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THE DISTRIBUTION OF POWER IN THE EUROPEAN CLUSTER GAME

INTRODUCTION

The weighted voting games are mathematical models which are used to analyze the distribution of the decision power of a nation in a supranational organization like the Council of Ministers of the European Union, the Security Council of the United Nations or the International Monetary Fund. In these institutions, each nation has associated a number of votes and a proposal is approved if a coalition of nations has enough votes to reach an established quota. The power of a country in a supranational organization is a numerical measure of its capacity to decide the approval of a motion. This decisive character is measured calculating the number of times that the vote of a country converts to a coalition that does not reach the quota to take decisions in a winning coalition. The power indices are a priori measures of this power, the most useful are the Shapley[1] and Banzhaf[2] indices.

Hagemann and De Clerck-Sachsse[3] obtained the next observation about the coalition formation in the European Union:

"But it can be concluded that a consistent pattern can be observed in the distinction between large, medium and small members; the following will reveal whether this differentiation also holds after the enlargement."

In this contribution, we study the European Cluster Game defined by the following six players:

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Big1 = {Germany},

Big2 = {France, United Kingdom, Italy},

Big3 = {Spain, Poland},

Med1 = {Romania, Netherlands, Greece, Portugal, Belgium, Czech Rep., Hungary},

Med2 = {Sweden, Austria, Bulgaria, Denmark, Slovakia, Finland, Ireland, Lithuania},

Small = {Latvia, Slovenia, Estonia, Cyprus, Luxembourg, Malta}.

The new voting rule proposed by the Intergovernmental Conference in order to draw up a future European Treaty changes in a very remarkable way the power of the countries in the Council. The reason is that the weighted votes, that were approved in Nice are removed and a coalition only needs 15 votes, which at least sum up by 65% of the population to approve a decision with the new rule. Furthermore, the minimum number of countries to block a proposal is four and the abstentions are not counted.

Cooperative games under combinatorial restrictions are cooperative games in which the players have restricted communication possibilities, which are defined by a combinatorial structure. The first model in which the restrictions are defined by the connected subgraphs of a graph is introduced by Myerson[4]. Contributions on graph-restricted games include Owen[5], and Borm, Owen, and Tijs[6]. In these models the possibilities of coalition formation are determined by the positions of the players in a communication graph. Another type of combinatorial structure introduced by Gilles, Owen and van den Brink[7] and van den Brink[8] is equivalent to a subclass of antimatroids. This line of research focuses on the possibilities of coalition formation determined by the positions of the players in the so-called permission structure.

We will analyze the European Cluster Game by using the restricted cooperation model derived from a combinatorial structure called augmenting system. This structure is a generalization of the antimatroid structure and the system of connected subgraphs of a graph. Furthermore, this new set system includes the conjunctive and disjunctive systems derived from a permission structure (see Bilbao[9]).

We will present a "ready-to-apply" procedure to compute the Shapley-Shubik index power of games restricted by combinatorial structures derived from the European Cluster Game. The second part of this contribution is devoted to the computation of the Owen index for the countries in the European Cluster Game. This game is a weighted multiple majority game with an a priori system of unions. We introduce the procedures to compute this power index by means of generating functions. Finally, we present the implementation of the algorithms used in this work in the computer system Mathematica by Wolfram[10].

AUGMENTING SYSTEMS

Let N be a finite set. A set system over N is a pair $(N, \mathcal{F})$ where $\mathcal{F}$ is a family of subsets of N. The sets belonging to $\mathcal{F}$ are called feasible. We will write $S \cup i$ and $S \setminus i$ instead of $S \cup \{i\}$ and $S \setminus \{i\}$ respectively.

Definition 1. *An augmenting system is a set system* $(N, \mathcal{F})$ *with the following properties:*
P1. $\emptyset \in \mathcal{F}$ ..
P2. *For* $S, T \in \mathcal{F}$ *with* $S \cap T \neq \emptyset$, *we have* $S \cup T \in \mathcal{F}$.
P3. *For* $S, T \in \mathcal{F}$ *with* $S \subset T$, *there exists* $i \in T \setminus S$ *such that* $S \cup i \in \mathcal{F}$.

Example. The following collections of subsets of $N = \{1,\ldots,n\}$, given by $\mathcal{F} \equiv 2^N$, $\mathcal{F} = \{\emptyset, \{i\}\}$ where $i \in N$, and $\mathcal{F} = \{\emptyset, \{1\}, \ldots, \{n\}\}$, are augmenting systems over N.

Example. In a communication graph $G=(N,E)$, the set system $(N,\mathcal{F})$ given by

$\mathcal{F}$ = { $S \subseteq N$: ($S,E(S)$) is a connected subgraph of G}, is an augmenting system.

The next characterization of the augmenting systems derived from the connected subgraphs of a graph is proved by Algaba, Bilbao, and Slikker[11].

Theorem 2. *An augmenting system* $(N,\mathcal{F})$ *is the system of connected subgraphs of the graph* $G=(N,E)$*, where* $E=\{S\in\mathcal{F} :: |S|=2\}$ *if and only if* $\{i\}\in\mathcal{F}$ *for all* $i \in N$ *.*

Example. Gilles et al. [10] showed that the feasible coalition system $(N,\mathcal{F})$ derived from the conjunctive or disjunctive approach contains the empty set, the ground set N, and that it is closed under union. Algaba et al.[12] showed that the coalition systems derived from the conjunctive and disjunctive approach were identified to poset antimatroids and antimatroids with the path property respectively. Thus, these coalition systems are augmenting systems.

Let $N = \{1,\ldots,n\}$ be a set of players with $n > 2$ and we consider a subset S of starting players. If $i \in S$ then the coalition $\{i\}$ is feasible. Each starting player i looks for a player $k \notin S$ to generate a new feasible coalition $\{i,k\}$. These coalitions with cardinality 2 searching for new players which agree to join one by one. If we assume that common elements of two feasible coalitions are intermediaries between the two coalitions in order to establish the feasibility of its union, we obtain an augmenting system $(N,\mathcal{F})$. Since the individual players $k \notin S$ are not feasible coalitions, the family $\mathcal{F}$ is not generated by the connected subgraphs of a graph. Moreover, if players $i,j\in S$ then $\{i\},\{j\}\in\mathcal{F}$ and $\{i,j\}\notin\mathcal{F}$. Then the augmenting system $(N,\mathcal{F})$ is not an antimatroid.

Example. Let $N = \{1,2,3,4\}$ and we consider $S1 = \{1,2,4\}$ and $S2 = \{1,4\}$. By using the above coalition formation model we can obtain the augmenting systems in Figure 1.

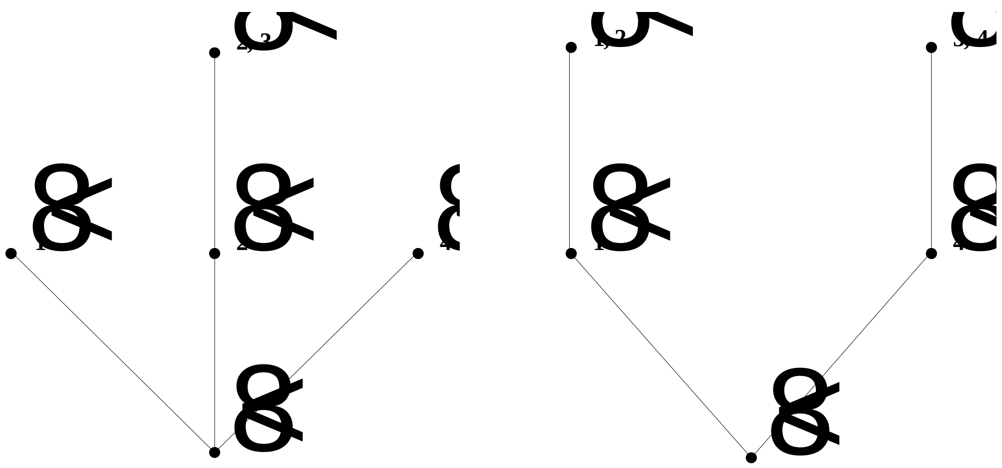

Figure 1

The sets of maximal feasible coalitions are partitions of the players into disjoint coalitions: the coalition structures $CS1 = \{\{1\},\{4\},\{2,3\}\}$ and $CS2 = \{\{1,2\},\{3,4\}\}$.

Example. Let us consider $N = \{1,2,3,4\}$ and

$$\mathcal{F} = \{\varnothing,\{1\},\{4\},\{1,2\},\{3,4\},\{1,2,3\},\{2,3,4\},N\}.$$

Since {1,2,3} and {2,3,4} are feasible coalitions, property P2 implies that the grand coalition N is also feasible (see Figure 2).

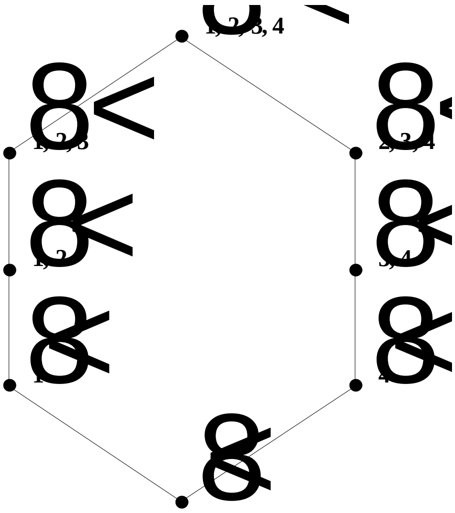

Figure 2

The set system given by $N = \{1,2,3,4\}$ and the family of subsets

$$\mathrm{F} = \{\varnothing,\{1\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},N\}.$$

is an augmenting system. Since $\{1,4\} \notin \mathrm{F}$ the system (N,F) is not an antimatroid. Moreover, $\{1,2\} \cap \{2,4\} = \{2\} \notin \mathrm{F}$ and so (N,F) is not a convex geometry (Figure 3).

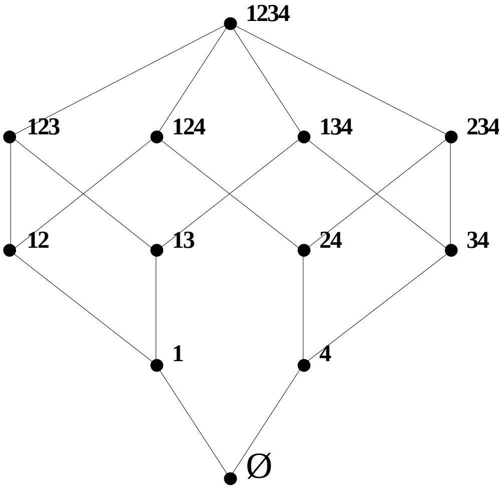

Figure 3

Definition 3. *Let* $(N,\mathcal{F})$ *be an augmenting system. For a feasible coalition* $S\in\mathcal{F}$ *, we define the set* $S^{*}=\{i\in N\setminus S : S\cup\{i\}\in\mathcal{F}\}$ *of augmentations of* S *and the set* $S^{+}=S\cup S^{*}=\{i\in N : S\cup\{i\}\in\mathcal{F}\}$.

Let $(N,\mathcal{F})$ be a set system and let $S\subseteq N$ be a subset. The maximal non-empty feasible subsets of S are called *components* of S. We denote by $C_{\mathcal{F}}(S)$ the collection of all components of a subset $S\subseteq N$. Observe that the set $C_{\mathcal{F}}(S)$ may be the empty set. This set will play a role in the concept of a game restricted by an augmenting system.

Proposition 4. *A set system* $(N,\mathcal{F})$ *satisfies property P2 if and only if for any* $S\subseteq N$ *with* $C_{\mathcal{F}}(S)\neq\emptyset$, *the components of S form a partition of a subset of S.*

GAMES RESTRICTED BY AUGMENTING SYSTEMS

Definition 5. *Let* $v:2^{N}\rightarrow\mathbb{R}$ *be a cooperative game and let* $(N,\mathcal{F})$ *be an augmenting system. The restricted game* $v^{\mathcal{F}}:2^{N}\rightarrow\mathbb{R}$, *is defined by*

$$v^{\mathcal{F}}(S)=\sum_{T\in C_{\mathcal{F}}(S)} v(T).$$

If $(N,\mathcal{F})$ is the augmenting system given by the connected subgraphs of a graph $G=(N,E)$, then the game $(N,v^{\mathcal{F}})$ is a graph-restricted game which is studied by Myerson [12] and Owen [14]. Note that if $S\in\mathcal{F}$ then $v^{\mathcal{F}}(S)=v(S)$. Let (N,v) be a game and $(N,\mathcal{F})$ an augmenting system. The *Shapley value* for player $i\in N$, in the restricted game $(N,v^{\mathcal{F}})$ is given by

$$\Phi_i\left(N,v^{\mathcal{F}}\right) = \sum_{\{S\subseteq N: i\in S\}} \frac{(s-1)!(n-s)!}{n!}\left[v^{\mathcal{F}}(S)-v^{\mathcal{F}}(S\setminus\{i\})\right],$$

where $n=|N|$ and $s=|S|$. This value is an average of the marginal contributions $v^{\mathcal{F}}(S)-v^{\mathcal{F}}(S\setminus\{i\})$ of a player i to the coalitions $\{S\subseteq N: i\in S\}$. In this value, the sets S of different size get different weight.

If the number of players is n, then the function that measures the worst case running time for computing these indices is in $O(n2^n)$. Moreover, to obtain the restricted game $\left(N,v^{\mathcal{F}}\right)$ we need to compute the set of the components $C_{\mathcal{F}}(S)$ of every subset $S\subseteq N$. Then it is necessary to consider all the feasible subsets of S and hence the time complexity is $O(t)$ where $t=\sum_{s=0}^{n}\binom{n}{s}2^s = 3^n$.

Bilbao[13] obtain the following explicit formula, in terms of v, for the Shapley value of the players in the restricted game $v^{\mathcal{F}}$. The time complexity of the formula is polynomial in the cardinality $|\mathcal{F}|$.

Theorem 6. *Let* $v: 2^N \to \mathbb{R}$ *be a cooperative game and let* $(N,\mathcal{F})$ *be an augmenting system. Then the Shapley value for player* $i\in N$, *in the restricted game* $\left(N,v^{\mathcal{F}}\right)$ *is*

$$\Phi_i\left(N,v^{\mathcal{F}}\right) = \sum_{\{T\in\mathcal{F}: i\in T\}} \frac{(t-1)!\left(t^+-t\right)!}{t^+!}v(T) - \sum_{\{T\in\mathcal{F}: i\in T^+\setminus T\}} \frac{t!\left(t^+-t-1\right)!}{t^+!}v(T),$$

where $t=|T|$ *and* $t^+=\left|T^+\right|$.

Remark. Notice that if $\mathrm{F} \equiv 2^N$, then $T^* = N \setminus T$ and $T^+ = N$ for every $T \in \mathrm{F}$. Thus, the formula obtained in the above theorem is equal to the classical Shapley value [15] for the game (N, v).

The algorithm showed in Theorem 6 computes the Shapley value $\Phi\left(N, v^{\mathrm{F}}\right)$ and written in the Mathematica computer system (Wolfram[14]) it is the following:

```
<<DiscreteMath`Combinatorica`
 Feasible[i_,F_List]:=Feasible[i,F]=Select[F,(MemberQ[#,i])&]
 SupInt[S_List,F_List]:=Select[T,(MemberQ[F,Union[S,{#}]])&]
 Augmentation[i_,F_List]:=Augmentation[i,F]=Select[F,
(DeleteCases[#,i]==#)&&(MemberQ[F,Union[#,{i}]])& ]
 co1[S_List]:=co1[S]=(Length[S]-1)!*
(Length[SupInt[S,F]]-Length[S])!/Length[SupInt[S,F]]!;
co2[S_List]:=co2[S]=co1[S]*Length[S]/
(Length[SupInt[S,F]]-Length[S]);
 RestrictedShapleyValue[game_:Null]:=Module[{value},
value=Table[Apply[Plus,If[#=={},0,
co1[#] (v[#])]& /@ Feasible[i,F]]-
Apply[Plus,If[#=={},0,co2[#] v[#]]& /@ Augmentation[i,F]],
{i,Length[T]}];Return[value]];
```

THE EUROPEAN CLUSTER GAME

The European Cluster Game is defined by the following six players which are bloks of countries with the corresponding population weights such that the total sum is 1000.

Big1 = {Germany (171)},

Big2 = {France (123), United Kingdom (123), Italy (118)},

Big3 = {Spain (86), Poland (79)},

Med1 = {Romania (45), Netherlands (33), Greece (23), Portugal (22), Belgium (21), Czech Rep. (21), Hungary (21)},

Med2 = {Sweden (18), Austria (17), Bulgaria (16), Denmark (11), Slovakia (11), Finland (11), Ireland (8), Lithuania (7)},

Small = {Latvia (5), Slovenia (4), Estonia (3), Cyprus (1), Luxembourg (1), Malta (1)}.

The voting method approved in the summit of Brussels on 18th June, 2004, for its incorporation to the European Constitution, is based on a double voting system and a blocking clause. To approve a proposal in the Council of Ministers of the 27 members of the European Union, it is needed at least 15 countries that sum up more or equal than 65% of the population. Moreover, the minimum number of countries to block a proposal is four and the abstentions are not counted. This game Cluster is defined as follows:

```
Cluster:= (T = Range[6]; Clear[w, z, p, q, v];
w[1]:= 171; w[2]:= 364; w[3]:= 165;w[4] := 186;
w[5]:= 99; w[6]:= 15 ;
p[S_List] := Plus @@ w /@ S;
z[1]:= 1; z[2]:= 3; z[3]:= 2; z[4]:= 7; z[5]:= 8; z[6]:= 6;
q[S_List] := Plus @@ z /@ S; v[{}] := 0;
v[S_ /; (p[S] >= 650 && q[S] >= 15) || q[S] >= 24] := 1;
```

v[S_ /; (p[S] < 650 && q[S] < 24) || q[S] < 15] := 0;)

We can also define the game Cluster1 given only by the double majority game without the blocking clause.

(v[{}] := 0; v[S_ /; (p[S] >= 650 && q[S] >= 15)] := 1;

v[S_ /; (p[S] < 650 || q[S] < 15)] := 0;)

The classical Shapley values of these games are:

ShaCluster = {0.10, 0.25, 0.10, 0.25, 0.15, 0.15},

ShaCluster1 = {0.067, 0.42, 0.067, 0.22, 0.12, 0.12}.

Let us consider the following star (Figure 4) and wheel (Figure 5) graphs.

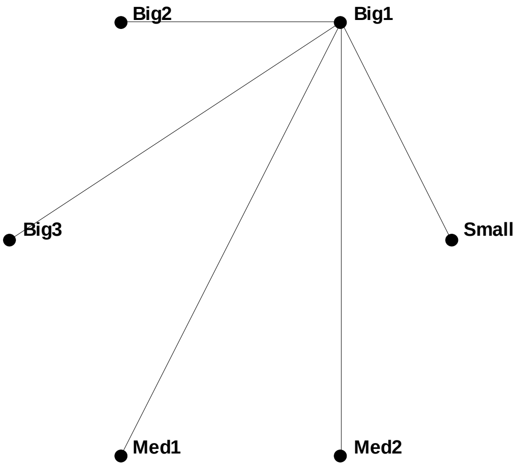

Figure 4

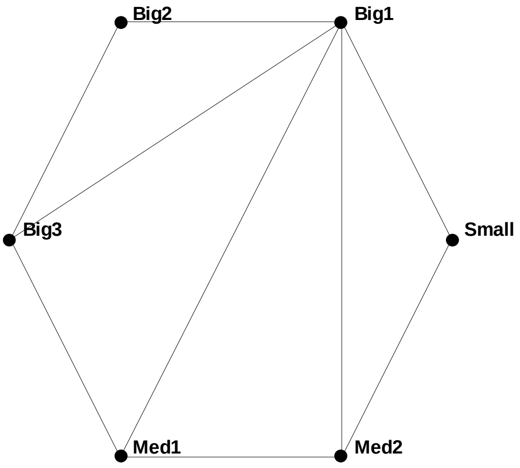

Figure 5

The augmenting system given by a graph *G* is the collection of all the connected subgraphs of *G*. For the above graphs, we obtain the following collections of feasible coalitions:

F_Star = {{}, {1}, {2}, {3}, {4}, {5}, {6}, {1, 2}, {1, 3}, {1, 4},

{1, 5}, {1, 6}, {1, 2, 3}, {1, 2, 4}, {1, 2, 5}, {1, 2, 6}, {1, 3, 4}, {1, 3, 5}, {1, 3, 6}, {1, 4, 5}, {1, 4, 6}, {1, 5, 6}, {1, 2, 3, 4}, {1, 2, 3, 5}, {1, 2, 3, 6}, {1, 2, 4, 5}, {1, 2, 4, 6}, {1, 2, 5, 6},

{1, 3, 4, 5}, {1, 3, 4, 6}, {1, 3, 5, 6}, {1, 4, 5, 6}, {1, 2, 3, 4, 5}, {1, 2, 3, 4, 6}, {1, 2, 3, 5, 6}, {1, 2, 4, 5, 6}, {1, 3, 4, 5, 6},

{1, 2, 3, 4, 5, 6}}

F_Wheel = {{}, {1}, {2}, {3}, {4}, {5}, {6}, {1, 2}, {1, 3}, {1, 4},

{1, 5}, {1, 6}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {1, 2, 3}, {1, 2, 4}, {1, 2, 5}, {1, 2, 6}, {1, 3, 4}, {1, 3, 5}, {1, 3, 6}, {1, 4, 5},

{1, 4, 6}, {1, 5, 6}, {2, 3, 4}, {3, 4, 5}, {4, 5, 6}, {1, 2, 3, 4},

{1, 2, 3, 5}, {1, 2, 3, 6}, {1, 2, 4, 5}, {1, 2, 4, 6}, {1, 2, 5, 6},

{1, 3, 4, 5}, {1, 3, 4, 6}, {1, 3, 5, 6}, {1, 4, 5, 6}, {2, 3, 4, 5},

{3, 4, 5, 6}, {1, 2, 3, 4, 5}, {1, 2, 3, 4, 6}, {1, 2, 3, 5, 6},

{1, 2, 4, 5, 6}, {1, 3, 4, 5, 6}, {2, 3, 4, 5, 6}, {1, 2, 3, 4, 5, 6}}

The Shapley value for the Cluster and Cluster1 games restricted for these graphs are respectively:

ShaStarCluster = {0.37, 0.17, 0.067, 0.17, 0.12, 0.12},

ShaStarCluster1 = {0.33, 0.33, 0.033, 0.13, 0.083, 0.083},

ShaWheelCluster = {0.17, 0.22, 0.12, 0.22, 0.17, 0.12},

ShaWheelCluster1 = {0.13, 0.38, 0.083, 0.18, 0.13, 0.083}.

Should be noted that the blocking clause changes the Shapley values of the European Cluster Game when we consider all feasible coalitions and also in the games restricted for the considered graphs.

THE OWEN VALUE OF GAMES WITH A PRIORI UNIONS

Let us consider a finite set *N*. We will denote by *P*(*N*) the set of all partitions of *N*. A partition $P \in P(N)$ is called a coalition structure or a system of unions of the set *N*. The Owen value, proposed and characterized by Owen[15], is an extension of the Shapley value for games with a priori unions. This value can be derived from a two-level bargaining process. First, unions split the total amount according to the Shapley value in the induced game played by the unions. Then, each union allocates its total reward among its members taking into account the possibilities that they might join another union using again the Shapley type of allocation.

For simple games with an a priori system of unions, a power index is a function which assigns to a simple game with an a priori system of unions (*N,v,P*) a vector *g*(*N,v,P*), where the *i*-component is the power of player *i* in the game (*N,v,P*) according to *g*. The *Owen index* can be written in this way:

$$g_i(N,v,P) = \sum_{L \subseteq M \setminus k} \sum_{T \subseteq P_k \setminus i} p^i_{L,T} \left(v\left(Q \cup T \cup i\right) - v\left(Q \cup T\right)\right),$$

for any $i \in N$, where $M = \{1,\ldots,m\}$, $P = \{P_1,\ldots,P_m\}$, $Q = \bigcup_{l \in L} P_l$, and P_k is the coalition of the partition *P* such that $i \in P_k$. For the Owen index the coefficient

$$p^{i}_{L,T} = \frac{l!(m-l-1)!}{m!} \frac{t!(p_k-t-1)!}{p_k},$$

which corresponds to the coefficients of the Shapley value in the games played by the unions and the members of the coalition P_k, respectively.

For large games, the computation of the Owen index needs a great number of operations and the computational complexity grows exponentially. The generating functions method is one of the procedures to compute this index. Generating functions give a procedure to enumerate the cardinality of elements *c*(*r*) of a finite set, when these elements have a configuration that depends on a characteristic *r*. These methods have been applied to compute the power of the countries in the International Monetary Fund by Alonso-Meijide and Bowles[16], and also in the European Union by Algaba, Bilbao, and Fernández[17].

We next consider the European Cluster Game as a game with the following a priori system of unions:

P1 = {Germany},

P2 = {France, United Kingdom, Italy},

P3 = {Spain, Poland},

P4 = {Romania, Netherlands, Greece, Portugal, Belgium, Czech Rep., Hungary},

P5 = {Sweden, Austria, Bulgaria, Denmark, Slovakia, Finland, Ireland, Lithuania},

P6 = {Latvia, Slovenia, Estonia, Cyprus, Luxembourg, Malta}.

The double majority voting system, as agreed in the 2004 IGC, will take effect on 1 November 2014, until which date the present qualified majority system (Nice rules) will continue to apply. After that, during a transitional period until 31 March 2017, when a decision is to be adopted by qualified majority, a member of the Council may request that the decision be taken in accordance with the qualified majority as defined in the Nice rules of the present Treaty of the European Union.

The weights with respect to the population and the number of countries in the double majority voting system are given by

Pop27 = {171,123,123,118,86,79,45,33,23,22,21,21,21,18,17,16,11,11,11,8,7,5,4,3,1,1,1};

Countries = {1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1};

where the quotas are 650 and 15 votes, respectively. Moreover, in the European Cluster Game we consider the following partition according to the population size of the countries of the European Union:

P = {{1},{2,3,4},{5,6},{7,8,9,10,11,12,13},{14,15,16,17,18,19,20,21},{22,23,24,25,26,27}};

We define the games Partition and Partition1 by using the double majority system with and without the blocking clause, respectively. In order to compute the Owen value of these games, we apply the generating functions algorithms given by Alonso-Meijide, Bilbao, Casas-Méndez, and Fernández[18]. The results obtained are:

OwenPartition = {0.0667, 0.139, 0.139, 0.139, 0.0417, 0.0250, 0.0333, 0.0306, 0.0306, 0.0306, 0.0306, 0.0306, 0.0306, 0.0161, 0.0161, 0.0161, 0.0137, 0.0137, 0.0137, 0.0137, 0.0137, 0.0194, 0.0194, 0.0194, 0.0194, 0.0194, 0.0194}

OwenPartition1 = {0.100, 0.0833, 0.0833, 0.0833, 0.0583, 0.0417, 0.0381, 0.0353, 0.0353, 0.0353, 0.0353, 0.0353, 0.0353, 0.0202, 0.0202, 0.0202, 0.0179, 0.0179, 0.0179, 0.0179, 0.0179, 0.0250, 0.0250, 0.0250, 0.0250, 0.0250, 0.0250}

The above results show the consequences of the blocking clause for the power of the european countries in the double majority voting system with the defined a priori system of unions. Germany, Spain and Poland are the only big countries than increase their power by using the blocking clause. In opposition, France, the United Kingdom, and Italy lose power. Furthermore, the 19 medium and small European countries win power, so that increase more power the small countries.

ACKNOWLEDGMENTS

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