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**ON THE DISTRIBUTION OF POWER
IN THE COUNCIL OF MINISTERS OF THE EU**

1. INTRODUCTION: THE COUNCIL OF THE EUROPEAN UNION

The European Parliament and the EU Council of Ministers are the two legislative institutions of the European Union. Although the power of the European Parliament has increased substantially over the last years and decades, the Council is still the main decision-making body of the Union. There are certain branches of politics, like agricultural affairs for example, which are still in complete control of the Council. In any other field the Council has at least as much influence as the Parliament.

The members of the Council are ministers of the member states, one from each state. Depending on the respective agenda each country will be represented by the minister responsible for the corresponding subject (foreign affairs, finance, social affairs, transport, agriculture, etc.).

It is clear that the procedure of decision-making in the council is of fundamental importance to the member states. In fact, it is crucial for the distribution of power in the Union. The bigger states like France or Germany certainly feel they should have a bigger influence than Luxembourg or Malta, for example. On the other hand, the smaller states fear to be dominated by the bigger countries, if the latter get too much power. It is therefore not

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surprising at all, that the distribution of power in the Council has been under heavy debate for a number of years at least.

The traditional decision procedure in the Council has been a weighted voting system. Each member state was given a certain number of votes (the voting weight) that had to be cast as a block. A proposal required a certain percentage of the total votes (usually about 70 %) for passing. This is known as “qualified majority voting”, in contrast to simple majority voting, which requires just more than 50%.

The voting weights of the member countries were not determined by a certain procedure or a mathematical formula (involving e.g. the country’s population or gross national product), but rather were the result of negotiations, frequently, in fact, of nightlong bargaining.

It was always tacitly accepted that a country with a bigger population should not have less votes in the Council than a country with a smaller population. On the other hand countries of roughly the same size always had the same number of votes. The economical power of a country has never been a criterion for its voting weight.

The table below shows the distribution of voting weights in the Council of the European Economical Community of 1958, one of the early predecessors of the current EU.

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Country	Voting weight	Population
Germany	4	About 60 Million
France	4	
Italy	4	
Netherlands	2	10 – 20 Million
Belgium	2	
Luxembourg	1	Less than 1 Million

Table 1: Voting weights in the EEC (1958)

It clearly distinguishes three groups of countries: The big countries (Germany, France and Italy), medium size countries (Netherlands and Belgium) and the small country Luxembourg. This voting system was extended without significant changes to the new member states of the EU in the various extension rounds of the Union (1973, 1981, 1986, 1995). The voting system in the EU of 15 valid until April 2004 is shown below.

Country	Voting weight	Population in million
Germany	10	62, later 82
France	10	60
U K	10	59

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Italy	10	57
Spain	8	40
Netherlands	5	16
Greece	5	11
Belgium	5	10
Portugal	5	10
Sweden	4	9
Austria	4	8
Denmark	3	5
Finland	3	5
Ireland	3	5
Luxembourg	2	0.5

Table 2: Voting weights for the Council from 1995 to 2004
 (Increase in German population due to unification)

It becomes clear from this table that now there are smaller big states and bigger medium size states etc. More precisely, the building of groups of states with comparable population size becomes increasingly difficult and, in fact, artificial.

The first voting reform for the Council was initiated at the European Summit in Nice 2000. At this meeting of the European Council the heads of state or government designed a three step voting procedure. In the first step a simple majority of states (each counted with weight 1) is required, the second step is a qualified majority voting (with modified voting

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weights similar to table 2), the third step is weighted voting with weights proportional to the population. This system is in effect since November 1st 2004. This voting procedure is certainly among the most complicated ever in use on this planet.

In the time period from February 2002 through July 2003 the European Convention designed a plan for a future Constitution of the European Union. This draft Constitution contains new rules for decision-making in the Council. The Convention essentially abandoned the second step of the Nice procedure, i.e. the qualified majority voting according to negotiated weights. The draft Constitution (in its original form) required a “double majority” for decisions of the Council. More precisely, for the approval of a proposal both the simple majority of the states and a qualified majority of weighted votes are required. The weights in the second step are distributed proportional to the population of the respective country and at least 60% of these weighted votes establish a qualified majority.

On the summit in Brussels in July 2004 the draft Constitution was changed by the leading European leaders. Besides minor changes, to be described below, they changed the simple majority of states to a 55% majority and the qualified majority of weights from 60% to 65%.

By 2005 the Constitution was ratified by 18 of the 27 member states, in most cases by the national parliaments, in Spain and Luxembourg by referenda. But on May 29 and June 1 the French and Dutch voters rejected the Constitution. This caused a halt in the ratification process. On three EU summits in 2007 the heads of states and governments discussed how to proceed with the institutional reform of the EU. They decided to replace the Constitution with a “Reform treaty” which in most member states will be ratified by the Parliament without a referendum.

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For most member states it seemed to be clear that the Reform treaty should adopt the “double majority” procedure for voting in the Council from the draft Constitution. Therefore, it appeared to come as a surprise when the Polish government pushed for another voting procedure, namely the “square root system”. In the square root system the voting weight for each state is proportional to the square root of the population, rather than proportional to the population number itself.

The Financial Times commented (FT June 11, 2007): “Their (i.e. the Poles’) slogan for the summit – ‘the square root or death’ – neatly combines obscurity, absurdity and vehemence...”. Many other newspapers joined the FT in its rejection of the Polish proposal.

In the following we will investigate and discuss various voting systems for the Council. Here we merely comment that the square root system could hardly be a surprise for the European leaders. It is based on results of L. Penrose from 1946 [12], it was proposed for the EU-Council by Swedish diplomats in the 90’s and in the paper [17], it was suggested to the EU-governments by about 50 scientists in an open letter to the governments and was discussed in various major newspapers and journals (for example the FAZ on June 11, 2004 and June 13, 2004, die Zeit 2004 [9], and the NZZ [14]). See also [1], [3], [7], [13], [18] as well as references given there.

Finally, the European Council implemented the “double majority” procedure for voting in the Council in 2014 (with a transition period until 2017). Until 2014 the voting system for the Council will be the “Nice” system.

The double majority indicates a paradigm shift, a drastic change of the system used so far. The voting weights are no longer assigned by bargaining in smoky backrooms. Moreover, it is completely clear how to extend the system whenever new members join the Union.

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The system looks pretty convincing. It reminds of the “Great Compromise” in the US-Constitution, the simple majority of states in the Senate and the proportional representation in the House of Representatives.

In the following we will analyse the system of Nice as well as the draft Constitution and compare both system with the ideal of “One person, one vote”. It will turn out that both voting system are pretty far from this democratic ideal.

2. A CLOSER LOOK AT A SIMPLE VOTING SYSTEM

Let us have a closer look at the voting system of the Council in the EEC (see table 1). Even at a first glance it is clear that the voting weights are not proportional to the populations. France, for example, has 150 times the population of Luxembourg, yet only four times its weight. So, one is tempted to believe that the small state Luxembourg has an disproportional big influence in the Council. However, the contrary is true.

Table 1 does not define the voting system completely. What is missing here is the quota, i.e. the minimal number of votes required for an affirmative decision. This quota was 12 (out of 17). 12 votes can be reached by either the three big states or by two big states and the two medium size states. In no case Luxembourg’s vote is needed to make a proposal pass. In other words: Luxembourg’s voting behaviour is totally irrelevant for decision-making in the Council of 1958.

We will later define power to be the ability of a voter (or a state) to influence the outcome of a decision through voting behaviour. In this sense, Luxembourg has no power at all.

It is instructive to investigate slight modifications of the system of Table 1. For example: Suppose, the quota was 11 instead of 12. Then a coalition of France, Italy, Belgium

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and Luxembourg, for instance, would be able to make a proposal pass. The same coalition without Luxembourg would not. So, Luxembourg has at least some power in the modified system. The same is true for a quota of 13.

The following modification of the system of '58 is even more astonishing: Let's increase the voting weight of the Netherlands from 2 to 3 (they are bigger than Belgium) keeping the quota at 12. This decreases the relative voting weight (the percentage) of the other states, the relative weight of Luxembourg from 5.8% to 5.5%.

Clearly, one expects that a *decrease* of the relative weight of a state will certainly *not increase* its power. But this is exactly what happens in this example. For instance, a coalition of Germany, Italy, Netherlands and Luxembourg would now have exactly 12 votes, so Luxembourg is able to influence the outcome of a decision in some cases.

There are two lessons to learn from this example: The quota plays a crucial role in a weighted voting system. Moreover, the voting weights, even together with the quota, do not directly give the distribution of power in that system. Of course, these data determine the power in a weighted voting system but only in a very complicated way.

The voting system discussed above is so simple that a close inspection of the weights and the quota allows us to analyse the system more or less completely. This is usually not so for systems with more voters, like in the present EU with 27 member states. In fact, in the Council with 27 members there are more than 134 million ways the different countries can vote with either "Yes" or "No".

It is the task of the next section to define exactly what we mean by "power" in a voting system and to develop tools to analyse it.

3. POWER IN VOTING SYSTEMS: THE PENROSE-BANZHAF MEASURE

Above we defined “power” as the ability of a voter to influence a decision by his or her voting behaviour. To make this more precise we consider a voting system with N voters. A subset of the voters is called a coalition. Such a set is called a winning coalition if affirmative votes of these voters can make a proposal pass. Otherwise a coalition is called losing. A voter v in a winning coalition C is called decisive (for C) if the defection of v from the coalition C makes it a losing coalition (i.e. C is winning, but C with v removed is losing). The more coalitions there are for which v is decisive, the more power v has. Thus we define the *Penrose-Banzhaf Measure of power* of a voter v to be the number of coalitions for which v is decisive divided by 2^{N-1} (which is a convenient normalisation).

For example, in the EEC-Council (table 1) there are six winning coalitions for which Belgium is decisive: Belgium together with either Germany (D), France (F) and the Netherlands (NL), or with F,I (Italy) and NL or with D,I and NL plus any of these together with Luxembourg. Hence the Penrose-Banzhaf Measure of Belgium is 6 divided by 32 which equals 0.1875. Note that e.g. in the winning coalition B, D, F, I Belgium is not decisive. A similar computation gives 0 for the Penrose-Banzhaf measure for Luxembourg (as we already expected), 0.1875 for NL and 0.3125 for D, F, and I.

In many cases we are mainly interested in the relative power of the voters, i.e. in the percentage of the total power of all voters. This relative power is given by the *Banzhaf Index* of voter v , which is the Penrose-Banzhaf Measure of v divided by the sum of the Penrose-Banzhaf Measures of all voters. In the above example the Banzhaf Index is 0.143 for B and NL and 0.288 for D, F, and I. Note that the Banzhaf Indices add up to one by definition. The Penrose-Banzhaf Measure contains more information than the Banzhaf Index. On the other

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hand the Banzhaf Index is easier to interpret when one compares the power of different countries. In the following we will usually use the Banzhaf Index to measure (relative) power.

Of course, it is impossible to consider 134 million possible coalitions by hand. However a PC will do the job for us. To do the computation we used both programmes written by ourselves and the freeware [4].

It is obvious that Power Indices cover only certain aspects of the complicated concept of power. The approach by Power Indices focuses completely on the power given by the technical procedure of voting in the given voting system. It ignores totally any informal influence like the power of the better argument, the power of better negotiation skills or of external threads. We are therefore fully aware that our notion of power covers but a small portion of the concept of power. However, we believe that being aware of its limitations it is *one* useful tool to analyse power in complex systems.

4. THE TREATY OF NICE

We start our investigation of the distribution of power in the EU-Council by an inspection of the treaty of Nice. This voting system was designed on the EU summit in Nice 2000. It is in effect since November 2004.

In Nice the leading European politicians designed a very complicated voting system which was advertised as a compromise between very different, actually contradictory demands of the participating countries. Germany for example wanted more power in the Council since its population jumped due to unification from 62 to 82 millions. France and UK certainly wanted to keep the same weight as Germany. Moreover, the small countries feared perhaps more than ever, a domination of the EU by the big countries. The Netherlands

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demanded more weight than e.g. Belgium, Spain tried to keep its status of a country with almost as much weight as Italy or France and so on.

It is almost a miracle, that the summit found a way to claim victory for all the diverging interests. First, the summit introduced a rule, that requires a simple majority of states. This was claimed to satisfy the demands of the small states. As a second step, the European leaders set up a weighted voting system with equal weights for Germany, France, UK and Italy (29) and a weight of 27 for Spain and Poland. This was celebrated as a victory for France as well as for Spain and Poland. The third rule was mainly introduced to pacify Germany's demand for a bigger influence. It is a weighted voting system with the country's population as the voting weight. Since all three requirements are necessary for a proposal to pass, all sides could claim a fair compromise.

Of course, it is not so easy to see what this complicated voting procedure really means. We have computed the Banzhaf Indices both for the EU of 25 and of 27. Some numbers are displayed in table 3. Our computations show that the first step – simple majority of states – has no effect at all. This is due to the fact that the quota for the second step is so high that it requires a simple majority automatically.

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	Population	Weight	Banzhaf Index
Germany	82,54	29	8,5606
France	59,63	29	8,5601
UK	59,09	29	8,5600
Italy	57,07	29	8,5600
Spain	40,68	27	8,1221
Poland	38,21	27	8,1221
Netherlands	16,19	13	4,23
Cyprus	0,80	4	1,3292
Luxembourg	0,45	4	1,3292
Malta	0,40	3	0,9933

Table 3: Some of the Banzhaf number for the Nice voting procedure
 (25 member states)

Moreover, we also observe that the third rule – voting by population size – has essentially no effect as well. For example Germany’s Banzhaf Index is only marginally bigger than the one of France although the population is bigger by one third. In fact, giving Germany 30 votes instead of 29 would have a much bigger effect on its Banzhaf Index than the whole third voting step.

Moreover, Spain and Poland – number 5 and 6 by population – could almost catch up with the “big four”. Poland’s population is roughly twice as big as the Dutch population and roughly half as big as the German one. Yet, Poland’s Banzhaf Index is almost twice as big as the one of Holland, but almost as big as the German Banzhaf Index.

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5. WHAT IS A DUE SHARE OF POWER FOR A COUNTRY?

In order to decide whether a voting system is fair or not we need an ideal reference to compare it with. In other words we need a criterion for a just voting system.

By any democratic standard it seems to be reasonable or even necessary that any EU-citizen should have the same influence on decisions of the Council regardless of his or her home country. It is likely that the European Convention had something like this in mind when they suggested the population criterion of the voting procedure. In deed, it sounds convincing that choosing weights proportional to the population implements equal influence for all citizens. In fact, most national parliaments try to represent the various regions of their country in this way.

However, there is an important difference between a parliament and the Council. In a parliament the people of a region are represented by a certain number of deputies who vote independent of each others— at least in principle. So, if 55% of the people in the region are in favour of a proposal, the percentage of deputies from that region favouring the proposal should also be about 55%. In contrast to this, the governments in the EU-Council cannot split their votes. So, if 55% of the Spanish population favours a proposal, the Spanish government has to cast *all* its votes in favour of the proposal. This reasoning already indicates that a number of votes proportional to the population might give too much influence to bigger countries.

To make a quantitative argument out of this reasoning we first consider the Penrose-Banzhaf Measure a citizen has in his or her country. It is clear that this number decreases as the number N of citizens in this country increases. It is not hard to see that the Penrose-Banzhaf Measure of a citizen is proportional to one over the square root of N . In fact, let us suppose N is odd, say $N=2n+1$. Then a voter in this country can change the outcome of a vote

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or referendum if the $2n$ other voters are split in two groups of equal size, i.e. if exactly n voters vote "Yes". There are $\binom{2n}{n}$ such coalition, so the Penrose-Banzhaf Measure of a voter in this country is $\binom{2n}{n}2^{-2n}$. For large N this is proportional to $\frac{1}{\sqrt{N}}$ by Stirling's formula.

So, to compensate this effect, the Penrose-Banzhaf Measure (and hence the Banzhaf Index) of the corresponding country in the Council should be proportional to the square root of N . This is the celebrated "Square root law" by Penrose. L. Penrose proved it as early as 1946 [12] (see also [6] and [11]).

The Polish suggestion, properly implemented, would exactly give this ideal voting system.

6. POWER INDICES FOR THE REFORM TREATY

The voting system set up by the European Convention has two big advantages over the previous systems:

- 1) It is relatively simple and easy to implement.
- 2) It is based on a simple formula to *compute* the voting weights, rather than on nightlong bargaining behind closed doors. This formula is easily extended whenever new members join the Union.

The only drawback is that the formula is *wrong*, i.e. it gives undue shares of power to the four big countries as well as to the small countries. While one may argue that the latter is a reasonable thing to do, there are good reasons to believe that the first effect is pretty dangerous for the future development of the European Union.

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Before we have a closer look at the figures here is a short table with a few countries. It displays the Banzhaf Indices for the draft Constitution in the Convention's version and the ideal indices according to the square root law.

Country	Pop.	SRL	Convention	Nice
Germany	82,54	9,54	12,76	7,78
France	59,63	8,11	9,09	7,78
UK	59,33	8,09	9,05	7,78
Italy	57,32	7,95	8,78	7,78
Spain	41,55	6,77	6,65	7,42
Poland	38,21	6,49	6,32	7,42
Latvia	2,33	1,60	1,68	1,25
Slovenia	2,00	1,48	1,64	1,25
Estonia	1,36	1,22	1,57	1,25
Cyprus	0,72	0,89	1,48	1,25
Luxemb.	0,45	0,70	1,46	1,25
Malta	0,40	0,66	1,44	0,94

Table 4: Comparison between Convention, Nice and the Square Root Law (SRL) (27 member states)

what is clearly visible from this table is that the four biggest countries all will considerably compared to the Nice treaty. While in the Nice treaty they got less power than described by the square root law, in the draft constitution (first version) they obtain more

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power than they should. Quite the opposite is true for Spain and Poland as well as for most middle size states.

In the light of this results it is not surprising that both Spain and Poland objected to the draft Constitution as suggested by the Convention. Reportedly, European leaders – in particular from Germany and Poland – tried to find a compromise by suggesting a change of the quota, namely to change the required majority of states from 50% to 55% and at the same time changing the “population”-quota from 60% to 65%. In essence, this became the finally version of the draft Constitution by decision of the European Council in Brussels July 2004.

A computation of the Banzhaf Indices shows that in this new version the share of power of Spain and Poland is even lower than in the original version if the Constitution (see table 5 in the Appendix). So, the 55 – 65 majority rule is certainly not a good compromise for those countries.

Table 6 in the Appendix shows that the relative losses in the draft Constitution (50-60 version) of most countries of medium size are quite large. Greece, Portugal, Belgium, the Czech Republic, Hungary and Sweden lose more than 20 % of their power compared to the square root law. For the 55-65 version of the draft Constitution these losses are still about 17 %.

It is of interest to try to estimate power indices according to the Constitution’s rule for further extension of the EU. If in the next decade Turkey will join the Union then Turkey and Germany will be the biggest states of the EU. In fact, due to its population increase Turkey will become more and more influential. For a discussion of this we refer to [5].

7. A MATHEMATICAL VIEW ON VOTING IN THE COUNCIL

In this section we sketch another more mathematical approach to find ideal voting weights. This section is based on the book [6] and the paper [11] which refined and extended the methods from [FM]. For details refer to these publications.

Our basic assumption is that representative democracy works, i.e. the governments know what the majority in their country wants and they act accordingly. Thus the representative of country A in the council votes "Yes" if (and only if) the majority in country A agrees with the proposal.

Suppose we have M states S_1, \dots, S_M with N_v voters in state S_v . Let us denote by X_{vi} the voting behaviour of voter i in state S_v . We set $X_{vi} = 1$, if this voter votes "YES", and $X_{vi} = -1$, if he or she votes "NO". Thus, the majority of voters in S_v agrees to a proposal if $X_v = \sum_{i=1}^{N_v} X_{vi} > 0$. The voting behaviour depends on the proposal ω under consideration, i.e. X_{vi} is a function $X_{vi}(\omega)$ of the proposal ω .

We denote the voting behaviour of state S_v in the council by Y_v . So, $Y_v = 1$ if

$X_v = \sum_{i=1}^{N_v} X_{vi} > 0$ and $Y_v = -1$ if $X_v = \sum_{i=1}^{N_v} X_{vi} \leq 0$. If state S_v has a voting weight g_v in the

Council the voting result in the council is $C = \sum_{v=1}^M g_v Y_v$.

From basic democratic principle it is desirable that the outcome of voting in the Council

agrees with the popular vote $P = \sum_{v=1}^M \sum_{i=1}^{N_v} X_{vi}$ of all voters in the union, i.e. we would like to

make the so called democracy deficit

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$$\Delta = \Delta(g_1, \dots, g_M) = |C - P| = \left| \sum_{v=1}^M \sum_{i=1}^{N_v} X_{vi} - \sum_{v=1}^M g_v Y_v \right|$$

as small as possible. There is no way to make Δ vanishing for all possible voting results: For any choice of the weights g_v one can find distributions of 1, -1 among the votes such that council vote and popular vote disagree. Moreover, Δ still depends on the proposal ω , so we can only hope to make Δ small "in the average". To define "in the average" rigorously we consider the proposal ω as random. This induces a probability structure on the set of all possible voting outcomes.

We suppose that the system is fed by "really random" proposals, in particular a proposal and its negative are equally likely. Consequently, the probability $P(X_{v_1 j_1} = a_1, \dots, X_{v_L j_L} = a_L)$ is the same as $P(X_{v_1 j_1} = -a_1, \dots, X_{v_L j_L} = -a_L)$ and $P(X_{v_j} = 1) = P(X_{v_j} = -1) = \frac{1}{2}$. Given the probability P we want to find voting weights g_1, \dots, g_v such that

$$(1) \quad E(\Delta^2) = E((C - P)^2)$$

is minimal, where E denote the expectation with respect to P.

If we assume that voters in state S_v make their decision independently of the voters in other states, it turns out that the optimal weight g_v minimizing (1) is given by

$$(2) \quad g_v = E \left(\left| \sum_{i=1}^{N_v} X_{vi} \right| \right).$$

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Equation (2) says that the optimal weight is given by the expected size of the majority in S_v .

We remark that $\sum_{i=1}^{N_v} X_{vi}$ is the number of "YES"-votes minus the expected number of "NO"-votes, i.e. the margin of the voting result in state S_v .

The next task is to compute $E\left(\left|\sum_{i=1}^{N_v} X_{vi}\right|\right)$. If we assume that the voters *inside* each country decide independently from each others then:

$$(3) \quad g_v = E\left(\left|\sum_{i=1}^{N_v} X_{vi}\right|\right) \approx \sqrt{N_v}$$

for a large number of voters. This is an easy consequence of the central limit theorem. Thus, we obtain again the *square root law*. The asymptotic (3) remains true as long as the central limit theorem hold, which is the case, roughly speaking, if the correlation between the voters is not too strong. Only, if we assume *strong* correlation then (3) has to be replaced by

$$(4) \quad g_v = E\left(\left|\sum_{i=1}^{N_v} X_{vi}\right|\right) \approx N_v^\alpha$$

for some α with $\frac{1}{2} < \alpha \leq 1$.

Models with correlation may serve to model a specific society at a certain moment and to make estimates based on this.

However, to establish a voting system in a constitutional act, we can not (and should not) reflect the momentary status of a particular society, but rather we should be led by general principles and long term considerations. For this purpose, the only reasonable basis seems to be the assumption of no (or weak) dependence between the voters.

For a discussion both of the mathematical aspects of these models and of the political implications see [11].

8. HOW TO COMPOSE A GOOD VOTING SYSTEM FOR THE COUNCIL

From time to time politicians and high rank EU officials claim that the voting system in the EU Council is not an important issue since most decisions are made unanimously. Given that the distribution of power was a key issue on various European summits and at least the summit in Rome failed just because of this very problem it is hard to believe that the question of power in the Council is not a major concern of the European politicians.

One can reasonably expect that some of the European politicians try to maximise the power of their respective countries. However, this is thinking in short time periods. An unjust voting system for the Council is very likely to block the further development of the European Union. Regardless of the short term winners of an unfair voting system all members of the Union will lose in the long run.

A fair voting system should not rely on "smoky backroom" negotiations of voting weights. It should rather be defined by a formula which gives voting weights (or better: voting power) on the basis of the countries' populations. In this respect, the draft Constitution is, indeed, a paradigm shift to the better. However, to choose voting weights proportional to the population gives the bigger states an undue share of power. Instead, the *power indices* of the states should be proportional to the square root of the respective population.

To implement such a system one could try to make the *voting weight* of a state with population N to be (proportional to) \sqrt{N} . In general this does *not* guarantee that the voting power is proportional to \sqrt{N} as well. However, it turns out that the voting weights in the EU-Council *are* proportional to \sqrt{N} if we choose the weights proportional to the square root *and*

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set the quota to 61,4 %. This is a result of the papers [15, 16] (see also [10]) by Słomczyński and Życzkowski, both from the Jagiellonian University at Krakow. The system they suggested is now known under the name "Jagiellonian Compromise"

One may argue that the EU is not only a union of people, but also a union of states. Therefore one can very well justify a "double majority". This means that in addition to a weighted voting (obeying the square root law) one might want to add a simple (or qualified) majority rule for the number of supporting states. This would strengthen the smaller states. Such a deliberate deviation from the square root law could turn out to be politically wise.

9. APPENDIX: COMPARISON OF THE VARIOUS VOTING SYSTEMS

The following table shows the Banzhaf Indices for the Council in a 27-member EU for the Nice treaty, the draft Constitution in the Convention's version (50-60) and in the current version (55-65), which will also be in the Reform Treaty . This is compared to the ideal distribution of power as given by the square root law (SRL)

Contry	Pop.	Nice	Constitution (50-60)	Constitution (55-65)	SRL
Germany	82,54	7,78	12,76	11,87	9,54
France	59,63	7,78	9,09	8,73	8,11
UK	59,33	7,78	9,05	8,69	8,09
Italy	57,32	7,78	8,78	8,44	7,95
Spain	41,55	7,42	6,65	6,38	6,77
Poland	38,21	7,42	6,32	5,89	6,49

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Romania	21,77	4,26	4,06	4,22	4,90
Netherl.	16,19	3,97	3,39	3,51	4,23
Greece	11,02	3,68	2,76	2,87	3,49
Portugal	10,41	3,68	2,69	2,80	3,39
Belgium	10,36	3,68	2,69	2,80	3,38
Czechia	10,20	3,68	2,66	2,78	3,36
Hungary	10,14	3,68	2,65	2,76	3,34
Sweden	8,94	3,09	2,50	2,62	3,14
Austria	8,07	3,09	2,40	2,52	2,98
Bulgaria	7,85	3,09	2,38	2,50	2,94
Denmark	5,38	2,18	2,07	2,19	2,44
Slovakia	5,38	2,18	2,07	2,19	2,44
Finland	5,21	2,18	2,04	2,17	2,40
Ireland	3,96	2,18	1,89	2,02	2,09
Lithuania	3,46	2,18	1,83	1,96	1,95
Latvia	2,33	1,25	1,68	1,81	1,60
Slovenia	2,00	1,25	1,64	1,78	1,48
Estonia	1,36	1,25	1,57	1,70	1,22
Cyprus	0,72	1,25	1,48	1,62	0,89
Luxemb.	0,45	1,25	1,46	1,59	0,70
Malta	0,40	0,94	1,44	1,58	0,66

Table 5

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The following table gives the *relative* gains or losses of the EU-member states compared to the square root law.

Country	Nice	Constitution (50-60)	Constitution (55-65)
Germany	-18,45	33,68	24,34
France	-4,05	12,07	7,67
UK	-3,81	11,84	7,44
Italy	-2,14	10,40	6,06
Spain	9,58	-1,84	-5,74
Poland	14,26	-2,67	-9,35
Romania	-13,10	-17,08	-13,89
Netherlands	-5,98	-19,73	-17,00
Greece	5,66	-20,84	-17,56
Portugal	8,69	-20,75	-17,35
Belgium	8,99	-20,53	-17,12
Czech Rep	9,80	-20,68	-17,23
Hungary	10,15	-20,80	-17,34
Sweden	-1,54	-20,38	-16,62
Austria	3,63	-19,51	-15,50
Bulgaria	5,08	-19,23	-15,15
Denmark	-10,52	-15,20	-10,04
Slovakia	-10,48	-15,16	-10,00

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Finland	-9,01	-14,80	-9,54
Ireland	4,32	-9,46	-3,29
Lithuania	11,57	-6,35	0,30
Latvia	-22,05	4,80	13,07
Slovenia	-15,74	10,80	19,72
Estonia	2,21	28,27	39,22
Cyprus	40,26	66,22	81,38
Luxembourg	77,76	107,12	126,34
Malta	42,30	118,15	138,56

Table 6

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