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DEGRESSIVE PROPORTIONALITY. COMPOSITION OF THE EUROPEAN PARLIAMENT. THE PARABOLIC METHOD

1. INTRODUCTION

The distribution of European Parliament (EP) seats among the member States of the European Union (EU) has been done so far through negotiation and agreements. A common principle, ever since the creation of the EU, has been to assign a greater representation to small States, and a lesser one to the big ones, than the representation that should correspond in proportion to population.

The Constitutional project of the EU states maintains this principle: it is set forth (article I-20[1]), that the seats should be distributed according to degressive proportionality, assigning a minimum of 6 representatives to any state no matter how small it is (such as the cases of Malta and Luxembourg, the two having a quota of less than one seat), and a maximum of 96, even ifit's the State's quota could be much greater than that (as happens nowadays with Germany). Also, the Parliament size must be equal to or less than 750 deputies.

Minimums and maximums for the EP similar to these have been established in all the EP elections in the past, which means that the requisites established in the project of the Constitution for calculating the representation of the EP States will lead to methods whose results should approximate the assignations reached in the past.

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Nevertheless, the project of the Constitution does not specify what means should be used to assign the seats to the States, and it does not afford a definition of "degressive proportionality". The last Intergovernmental Conference, in July 2007, established the same three limitations as the project of the Constitution, and also succinctly described degressive proportionality, but the definition given could prove non-applicable sometimes, or could lead to several widely varying solutions even at present. The application of degressive proportionality in cases that have several solutions calls for agreements, which implies a system where negotiating efficacy and State power predominate, and which tends to produce precarious results from the viewpoint of the stability and legitimacy of the system. The alternative consists in introducing, in terms of logical reasoning, a mathematical formula that could accommodate the principle of degressive proportionality and the minimum and maximum limitations.

Accordingly, a number of works have been presented in specialized conferences on Electoral Systems and have been published in several journals[2,3,4,5,6,7]. In this paper, a new property for degressive proportionality, *degressive quota variation,* is introduced. In agreement with these properties, several methods related to EP seat distribution, in the context of the Constitution project, have been obtained. Highlighted among them is *the parabolic method*, as it is one of the simplest methods to apply, it is more convenient regarding the matter of degressive proportionality, and it is adaptable, so as to accommodate changes in the number of States that make up the EU or in their populations, as well as changes in the minimums and maximums established in the Constitution project. This revision does not modify the results of the parabolic method studied in previous works, yet it does contribute with new properties that enhance and strengthen the concept of degressive proportionality.

To this end, Section 2 deals with the report of Lamassoure and Severin[8] (LS) from the Commission of Constitutional Affairs of the European Parliament, which contains a proposal for the composition of the EP for the period 2009-2014; and it seen that the concept of degressive proportionality as established by the Intergovernmental Conference and used by both Euro MPs to elaborate the report is not always applicable. We then set forth three reasonable properties for degressive proportionality. In section 3, the parabolic method is described, and it is applied to determine the composition of the EP using the current data and for an enlargement of the EU if Croatia is included. Section 4 offers a distribution of 750 seats in proportion to the square root of the population between the 27 states of our current EU, demonstrating that the square root method does not ensure some of the basic characteristics that we have established for degressive proportionality. Finally, some conclusions are drawn.

2. MATHEMATICAL FOUNDATIONS OF DEGRESSIVE PROPORTIONALITY

2.1 EARLY ATTEMPTS TO DEFINE DEGRESSIVE PROPORTIONALITY: AN IMPOSSIBLE REQUIREMENT!

For several years we have been using the term degressive proportionality without rigorously defining the concept. Always present was the notion that the smallest States should receive more seats than they were proportionally allotted, whereas the opposite should be true of the larger states; and furthermore, one given State could not have more representatives than another with a greater population. In the past, negotiations have verified this dual criterion. Yet we still need further clarification of degressive proportionality.

More recently, a rigorous definition has arisen for the term degressive proportionality, appearing both in the Lamassoure/Severin report and in the article by James-Chopin[9]. Degressive proportionality is indicated to signify that the quotient between the population of

each State and its number of deputies or representatives should increase along with increasing population. This means that the ratio between the exact quota of a given State and the number of representatives should likewise decrease along with a decreasing population.

The initial objection to this definition resides in the fact that it can be verified through very widely different allotments. For instance, on the basis of the population figures used to determine the composition of the European Parliament for the period 2009-2014, the two distinct distributions of 750 seats shown in Table I (A1 and A2) both come to verify this property.

Table I. Two very different allotments verifying Lamassoure/Severin criterion

Member State	**Population**	**Exact Quota**	**A1**	**EQ/A1**	**A2**	**EQ/A2**
Malta	404,300	0.62	**6**	9.752	**6**	9.752
Luxembourg	459,500	0.70	**6**	8.581	**6**	8.581
Cyprus	766,400	1.17	**6**	5.145	**10**	8.575
Estonia	1,344,700	2.05	**6**	2.932	**16**	7.819
Slovenia	2,003,400	3.05	**6**	1.968	**16**	5.248
Latvia	2,294,600	3.49	**6**	1.718	**16**	4.582
Lithuania	3,403,300	5.18	**8**	1.545	**16**	3.090
Ireland	4,209,000	6.40	**9**	1.405	**16**	2.498
Finland	5,255,600	8.00	**11**	1.375	**17**	2.126
Slovakia	5,389,200	8.20	**11**	1.341	**17**	2.073
Denmark	5,427,500	8.26	**11**	1.332	**17**	2.058
Bulgaria	7,718,800	11.75	**15**	1.277	**17**	1.447

Austria	8,265,900	12.58	**16**	1.270	**17**	1.351
Sweden	9,047,800	13.77	**17**	1.235	**17**	1.235
Hungary	10,076,600	15.33	**18**	1.174	**17**	1.109
Czech Rep	10,251,100	15.60	**18**	1.154	**17**	1.090
Belgium	10,511,400	15.99	**18**	1.125	**17**	1.063
Portugal	10,569,600	16.08	**18**	1.119	**17**	1.057
Greece	11,125,200	16.93	**18**	1.063	**17**	1.004
Netherlands	16,334,200	24.85	**25**	1.006	**21**	0.845
Rumania	21,610,200	32.88	**32**	0.973	**27**	0.821
Poland	38,157,100	58.06	**55**	0.947	**47**	0.809
Spain	43,758,300	66.58	**62**	0.931	**53**	0.796
Italy	58,751,700	89.40	**83**	0.928	**71**	0.794
United K.	60,421,900	91.94	**85**	0.924	**72**	0.783
France	62,886,200	95.69	**88**	0.920	**74**	0.773
Germany	82,438,000	125.44	**96**	0.765	**96**	0.765
Total UE-27	492,881,500	750.00	**750**		**750**	

In the first allotment, A1, Estonia gets six representatives, whereas under allotment A2 Estonia obtains 16 representatives. Contrariwise, France is given 88 representatives under A1, yet just 74 with A2. Although there is considerable discrepancy between the two allotments for a number of States, under both systems Malta gets six deputies and Germany receives 96.

A very distinct allotment that those of Table 1 can be obtained with a nearly egalitarian allotment; for example, we might assign 28 seats to each member State with the exception of the six smallest (Malta to Latvia), which would obtain 27 each, so as to arrive at

a total of 750. And this allotment, nearly identical for all the States involved, verifies the established property.

Therefore, despite the fact that degressive proportionality is defined with precision, there may exist many allotments that fulfil the requirements while producing widely varying results.

The second matter at hand is even more relevant. Will there always be at least one distribution of the seats of Parliament that verifies this property? Unfortunately, the answer is NO. That is, there are situations in which *it is impossible to reach an allotment* that verifies the condition established in the Lamassoure/Severin report. Let us have a look at two very simple examples that can be easily checked.

INAPPLICABLE DEFINITION OF LAMASSOURE/SEVERIN

EXAMPLE 1

We show a very simple case, using a number of the current States with the corresponding populations that must be used to determine the Composition of the EP for the period 2009-2014. These figures appear in Table II, which contains six of the 27 current States and assumes a Parliamentary size of 116 seats (those that the parabolic method assigns at present, although other sizes for the house of deputies would also be valid for our purposes).

Table II. Inapplicable Lamassoure/Severin criterion. H=116

State	**Population**	**LS-Allotment**
Greece	11,125,200	
Portugal	10,569,600	

Belgium	10,511,400	**Impossible**
Czech Rep.	10,251,100	
Hungary	10,076,600	
Switzerland	9,047,800	
Total	61,581,700	116

In view of this example, it is not rational to accept for "degressive proportionality" what is stipulated in point 6 of the LS report, forced by the mandate of the Intergovernmental Conference, which on occasions demanded a property impossible to fulfil, as shown above.

EXAMPLE 2

Another example that is impossible to solve is the one appearing in Table III. In this case, we have substantially increased the population of two of the States, making the verification immediate, as with these populations in order to arrive at a solution verifying the condition used by LS, the size of the EP must be even.

Table III. Inapplicable Lamassoure/Severin criterion. H=255

Minimum = 2 or 3 or 4 or 5 or 6; Maximum = 70 or … or 96

State	**Population**	**LS Allotment**
Malta+50.000	454,300	
Luxembourg	459,500	
Slovakia	5,389,200	
Denmark	5,427,500	
Hungary	10,076,600	**IMPOSSIBLE**

Czech R	10,251,100	
Belgium	10,511,400	
Portugal	10,569,600	
United K. +2M	62,421,900	
France	62,886,200	
Total EU-27	178,447,300	255

Including all 27 States, it is also possible to provide examples of populations in such a way that there is no distribution verifying the property of degressive proportionality in the established sense. This fact is more difficult to prove, requiring slight changes in the population of some States.

It is feasible that such a situation could exist for any given particular distribution. Yet imposing a decrease in the quota/seat ratio entails another important consequence: *Statesthat differ little in population are forced to receive an equal number of deputies*. Hence, in a chain of States whose populations differ little, upon passing from one to the following one, all must receive the same number of deputies. This is, however, probable enough, and in fact is precisely what has happened with all the States included among Greece and Hungary as a result of the LS proposal.

As a consequence, a new and specific definition should be granted to the term degressive proportionality. This is the point made in the following section.

2.2. DEGRESSIVE PROPORTIONALITY

This section is intended to briefly describe the mathematical details behind the justification of degressive proportionality.

2.2.1. Exact quotas and methods of proportional apportionment

The *quota* or *exact quota* of every member State is the exact fraction of seats that corresponds to it in proportion to its population (the aforementioned fraction appears in the previous tables rounded to two decimal figures). A proportional assignment can always be carried out from the quotas, using a method of proportional distribution. Such proportional methods include those of d'Hondt (or Jefferson), Webster (or Sainte-Laguë), Greater Remainders (or Hamilton), etc. If we wish the method of distribution to be unbiased, consistent, and monotonous, the most advisable method is that of Webster[10]. This is the reasoning behind our use of Webster's method in the approximation of integer numbers in this paper.

2.2.2. Properties for degressive proportionality, and limitations

When representation need not be proportional, but be endowed with degressive proportionality (as required by the European Constitution project, and the Intergovernmental Conference as well), in order to distribute the EP seats among the States of the Union, one way of raising it is by using a function *A* that transforms the exact quotas "*q*" into other quantities *A*(*q*), called adjusted quotas. Let us now look at the implications of such a function of adjustment.

1. *Growth.* The function of adjustment of the quotas, *A*, may not be a decreasing function, since a State having more inhabitants than another must not receive a lesser number of representatives. That is to say, if q_i and q_j are the exact quotas from two member States *i* and *j*, and $A(q_i)$ and $A(q_j)$ the readjustments thereof, then

$$q_i < q_j \quad \Rightarrow \quad A(q_i) \leq A(q_j) \qquad (1)$$

2. *Degressive variation of the quota.* The largest States will lose representation upon the modification of their exact quota into adjusted quotas, whereas the smallest ones will see an increase in representation. It would seem logical that the smaller the State, the more it gains with the readjustment of its quota. Particularly, if:

$$q_i < q_j \quad \Rightarrow \quad A(q_i) - q_i \geq A(q_j) - q_j \qquad (2)$$

and therefore, the difference between adjusted quotas and exact quotas must involve a decrease when the population increases.

3. *Degressive ratio between adjusted quota and exact quota.* The ratio $\frac{A(x)}{x}$ must be decreasing (very similar to the Lamassoure/Severin criterion, but not the same). The smallest States gain more in relative terms; that is, the quotient between the adjusted quota and the exact quota decreases along with the population.

$$q_i < q_j \quad \Rightarrow \quad \frac{A(q_i)}{q_i} \geq \frac{A(q_j)}{q_j} \qquad (3)$$

A number of adjustment functions can verify the three properties stated above. The graphs may be rectilinear or even horizontal.

The 1st, 2nd and 3 rd properties mean that transform *A* is degressively proportional.

Now for the limitations regarding the European Parliament.

4. *To award H seats.* With the adjustment for degressive proportionality, the smallest States will increase their quota whereas the quota of the large ones will diminish. The sum of the adjusted quotas, rounded to the nearest integer number, has to go on coinciding with the number of seats to be distributed, that is to say, if q_i, are the exact quotas, $A(q_i)$ the readjustments of those same ones, and $\left[A(q_i)\right]_W$ the rounding of the quantity $A(q_i)$ to the nearest integer number (we suppose that Webster's method is used for rounding) , then

$$\sum_{i=1}^{n}\left[A(q_i)\right]_W = H \qquad (4)$$

Here, H is the size established for the European Parliament, n the number of States.

Finally, we still have to decide on the minimum and maximum limits. It would appear logical for those limits to be established in the project of the Constitution, with the intention that the representation of the smallest State either coincide with or be next to the minimum, if possible; and that the number of seats of the most populous State coincides with the maximum, or be as near as possible to it. For this reason we shall include a final condition for the function of adjustment to determine the composition of the EP.

5. *Limitations in the ends of the interval.* If m is the smallest quota (belonging to Malta in our case) and M is the biggest quota (corresponding to Germany), it is desirable to require that the function of adjustment verify

$$\left.\begin{array}{l} A(m) = 6 \\ A(M) = 96 \end{array}\right\}.$$

But, this condition may prove impossible to fulfil in same cases. For instance, it suffices to imagine a size of Parliament for which the quota of the biggest State is less than 96; or else the quota of the smallest one is greater than 6. In either of these two cases (and in many other cases), there exists no function beginning at 6 and ending in 96 and that will verify the first four properties.

Therefore, we shall consider as a fifth condition that:

> If possible, the limitations at both ends of the interval be reached; and when this is not possible, one of two conditions will have to be forced and the other will be approximated to the maximum. (5)

Remarks:

1st. If the function of adjustment is derivable in the interval $[m,M]$, then it is sufficient to verify that $A'(x) \geq 0,\ \forall x \in [m,M]$ in order for growth to be guaranteed.

2nd. On the other hand, (2) implies that rounding using the Webster method must verify

$$q_i < q_j \quad \Rightarrow \quad \left[A(q_j)\right]_W - \left[A(q_i)\right]_W \leq q_j - q_i + 1. \qquad (2')$$

3rd. If A is derivable, a condition that suffices to guarantee property (2) is for its derivative not to be superior to one. Then, if $0 \leq A'(x) \leq 1,\ \forall x \in [m,M]$, properties (1) and (2) are guaranteed.

With regard to the third property, we observe that the approximation to entire quantities of the adjusted quotas does not necessarily have to continue verifying such, since

$q_i < q_j$ implies $\dfrac{A(q_i)}{q_i} \geq \dfrac{A(q_j)}{q_j}$, but it does not imply $\dfrac{\left[A(q_i)\right]_W}{q_i} \geq \dfrac{\left[A(q_j)\right]_W}{q_j}$

(Moreover, this very last condition, together with growth, is impossible to fulfil under certain circumstances, as we will see in Section 6)

3. THE PARABOLIC METHOD.

A function of adjustment of the quotas is of the parabolic type when it is expressed in the following way:

$$A(x) = a + bx + cx^2, \quad x \in [m,M].$$

The graph of a curve of this type is a parabola of the second degree (it is a straight line when c is zero).

Further on, to clarify things a bit, we will be using the limitations that appear in the project of the Constitution —that is, 6 and 96— when referring to size H of the European Parliament, and it will always be understood that H is a size compatible with the limitations established. H must verify that $6n \leq H \leq 96n$.

3.1. THE PARABOLIC METHOD USING CURRENT DATA

For the current data and the requirements of Parliament size H=750, minimum 6 seats and maximum 96 seats for each member State (using Webster for rounding), all parabolas

$$A(x) = 6 + \frac{90}{M-m}(x-m) - cx^2$$
$$= 6 + \frac{90}{125.44-0.62}(x-0.62) - cx^2, \quad c \in [0.00143257, 0.00150033]$$

give the same allotment, and all verify the five properties described above in Section 2.2.2. The allotment corresponding to the parabolic method for the current 27 Member States appears in the column 'Parabolic' of Table III. One of the parabolas is

$$A(x) = 5.44084 + 0.909819x - 0.00149792x^2$$

Table IV. Parabolic allotment and Lisbon proposal for the EP

State	**Exact Quota**	**Ajust. Quota**	**AQ-EQ**	**AQ/EQ**	**Parabolic**	**Lisbon Proposal**
Malta	0.62	6.00	5.38	9.75	**6**	6
Luxembourg	0.70	6.08	5.38	8.69	**6**	6
Cyprus	1.17	6.50	5.33	5.57	**6**	6
Estonia	2.05	7.30	5.25	3.57	**7**	6
Slovenia	3.05	8.20	5.15	2.69	**8**	8
Latvia	3.49	8.60	5.11	2.46	**9**	9
Lithuania	5.18	10.11	4.93	1.95	**10**	12
Ireland	6.40	11.21	4.81	1.75	**11**	12
Finland	8.00	12.62	4.62	1.58	**13**	13

Slovakia	8.20	12.80	4.60	1.56	**13**	13
Denmark	8.26	12.85	4.59	1.55	**13**	13
Bulgaria	11.75	15.92	4.17	1.36	**16**	18
Austria	12.58	16.65	4.07	1.32	**17**	19
Sweden	13.77	17.68	3.91	1.28	**18**	20
Hungary	15.33	19.04	3.71	1.24	**19**	22
Czech Rep.	15.60	19.27	3.67	1.235	**19**	22
Belgium	15.99	19.61	3.62	1.226	**20**	22
Portugal	16.08	19.69	3.61	1.224	**20**	22
Greece	16.93	20.41	3.48	1.21	**20**	22
Netherlands	24.85	27.13	2.28	1.09	**27**	26
Rumania	32.88	33.74	0.86	1.03	**34**	33
Poland	58.06	53.22	- 4.84	0.92	**53**	51
Spain	66.58	59.38	- 7.20	0.89	**59**	54
Italy	89.40	74.81	-14.59	0.84	**75**	73
United K.	91.94	76.43	- 15.51	0.83	**76**	73
France	95.69	78.79	- 16.90	0.82	**79**	74
Germany	125.44	96.00	- 29.44	0.77	**96**	96
Total UE-27	750.00	750.00			**750**	751

Table IV shows the composition of the European Parliament obtained applying the parabolic method. In the 2nd column, the exact proportion (rounded to hundredths) of deputies that would correspond to each State is shown. These are the exact quotas. In the 3rd column we show the result of readjusting these quotas using a parabolic function (the adjusted

quotas). The adjusted quotas are increasing (Property 1). The 4th column contains the loss or gain in quota for each State: "*A more populous member State than another loses more quota than the less populous one*" (Property 2). The 5th column gives the quotient between the adjusted quotas and the exact quotas: "*As the population grows, the quotient between adjusted quotas and exact quotas decreases"* (Property 3). The 6th column contains the distribution using the parabolic method, that is, rounding off to the nearest integer the number of the adjusted quota of each State. The size of the European Parliament is 750 seats and the population of each State is the same as in Table I, which is the one that appears in the Eurostat (European Communities Statistics Office) database corresponding to the year 2006[11], and is the one recommended for use according to the Bulletin of the European Union, 4.1.2007. Finally, the 7th column contains the proposal presented at the Lisbon Conference on October 19, 2007.

If we wish to compare the distribution obtained under the parabolic method with that of the Lisbon proposal for the composition of the European Parliament in 2009-2014, we arrive at the following observations.

Remarks:

1. The Lisbon proposal does not verify some of its own principles:
    a. The parliament size is 751; therefore this size is greater than 750.
    b. The price of Spanish deputies is 433,758,300/54 = 810,339 and the price for the Italian deputies is 58,751,700/73 = 804,818, less than the Spanish price.
2. The Lisbon proposal does not verify property 2 relative to *degressive variation of the quota*. We see that condition (2') fails for several comparisons, for example:
    a. The difference in quota between Lithuania and Latvia is 5.18 – 3.49 = 1.69, hence Lithuania should not obtain 3 deputies more than Latvia.

b. The difference in quota between Bulgaria and Denmark is 11.75 – 8.26 = 3.49, so Bulgaria should not obtain 5 deputies more than Denmark.

c. The difference in quota between Hungary and Slovakia is 15.33 – 8.20 = 7.13, so Hungary should not obtain 9 deputies more than Slovakia.

And so on.

Graphically, the allotment with the parabolic method and the Lisbon proposal can be seen in Figures 1 and 2.

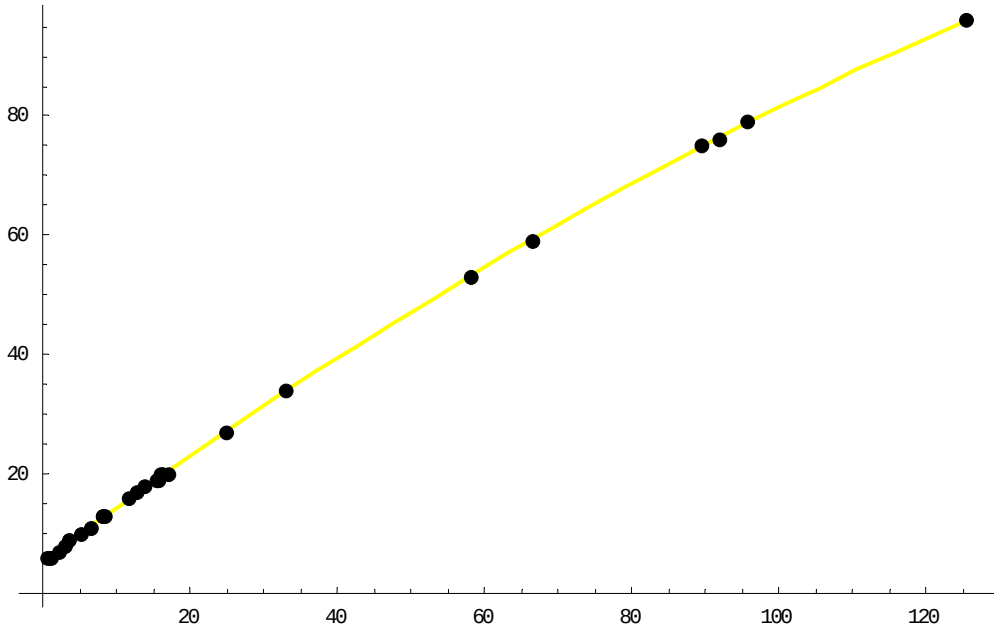

FIGURE 1: Parabolic allotment and adjusting function for H = 750 and n = 27.

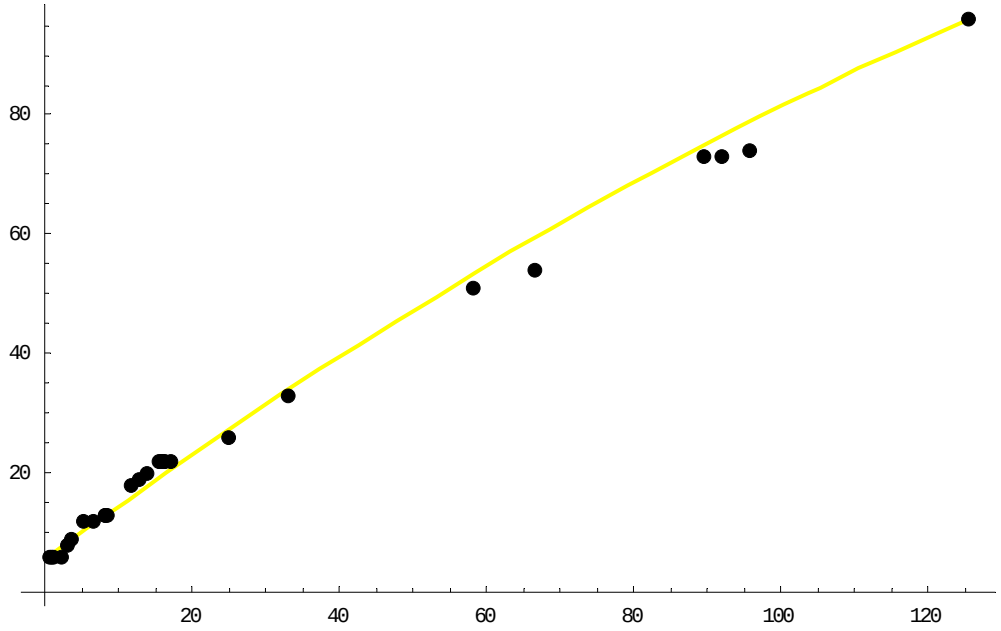

FIGURE 2: Lisbon proposal (points) and parabolic adjusting curve.

3.2 PARABOLIC METHOD WITH THE INCORPORATION OF CROATIA

In this case, one parabola obtained is

$$A(x) = 5.46257 + 0.882204x - 0.001233836x^2$$

and the parabolic allotment is shown in Table V.

Table V. Parabolic allotment for EU28=EU27+Croatia

State	**Population**	**Exact Quota**	**Ajust. Quota**	**AQ-EQ**	**AQ/EQ**	**Parabolic**
Malta	404,300	0.61	6.00	5.39	9.841	6
Luxembourg	459,500	0.69	6.07	5.38	8.764	6
Cyprus	766,400	1.16	6.48	5.32	5.607	6
Estonia	1,344,700	2.03	7.25	5.22	3.573	7
Slovenia	2,003,400	3.02	8.12	5.10	2.687	8
Latvia	2,294,600	3.46	8.50	5.04	2.457	9

Lithuania	3,403,300	5.13	9.96	4.83	1.940	10
Ireland	4,209,000	6.35	11.01	4.66	1.735	11
Croatia	4,442,800	6.70	11.32	4.62	1.689	11
Finland	5,255,600	7.93	12.38	4.45	1.561	12
Slovakia	5,389,200	8.13	12.55	4.42	1.544	13
Denmark	5,427,500	8.19	12.60	4.41	1.539	13
Bulgaria	7,718,800	11.64	15.56	3.92	1.337	16
Austria	8,265,900	12.47	16.27	3.80	1.305	16
Sweden	9,047,800	13.64	17.27	3.63	1.266	17
Hungary	10,076,600	15.20	18.58	3.38	1.223	19
Czech Rep.	10,251,100	15.46	18.80	3.34	1.216	19
Belgium	10,511,400	15.85	19.14	3.29	1.207	19
Portugal	10,569,600	15.94	19.21	3.27	1.205	19
Greece	11,125,200	16.78	19.92	3.14	1.187	20
Netherlands	16,334,200	24.63	26.44	1.81	1.073	26
Rumania	21,610,200	32.59	32.90	0.31	1.009	33
Poland	38,157,100	57.54	52.13	- 5.41	0.906	52
Spain	43,758,300	65.99	58.29	- 7.70	0.883	58
Italy	58,751,700	88.60	73.91	- 14.69	0.834	74
United K.	60,421,900	91.12	75.57	- 15.55	0.829	76
France	62,886,200	94.84	77.99	- 16.85	0.822	78
Germany	82,438,000	124.32	96.00	- 28.32	0.772	96
Total EU-27	497,324,300	750.00	750.00			750

In this case we are going to suppose that the European Union has been extended to 28 members by Croatia's entry (it might possibly be the next State to be incorporated). The simulation is done for an EP size of 750 seats and the limitations 6 and 96. The results obtained with the parabolic method are those shown in the last column of Table V.

Compared with the contents of Table IV, Croatia receives 11 representatives. This is only possible if 11 of the 27 current States lose one representative each with regard to the assignment that appears in Table IV. Again, it is important to note that the quantities that appear in the column AQ-EQ and in the column AQ/EQ decrease as the population grows. In this case, as in Table IV, the six most populous States are the only that lose quota, whereas the 22 remaining ones gain. Both in absolute terms and in relative terms, the smaller the State, the more it gains with the adjusted parabola.

Graphically, the allotment with the incorporation of Croatia is seen in Figure 3.

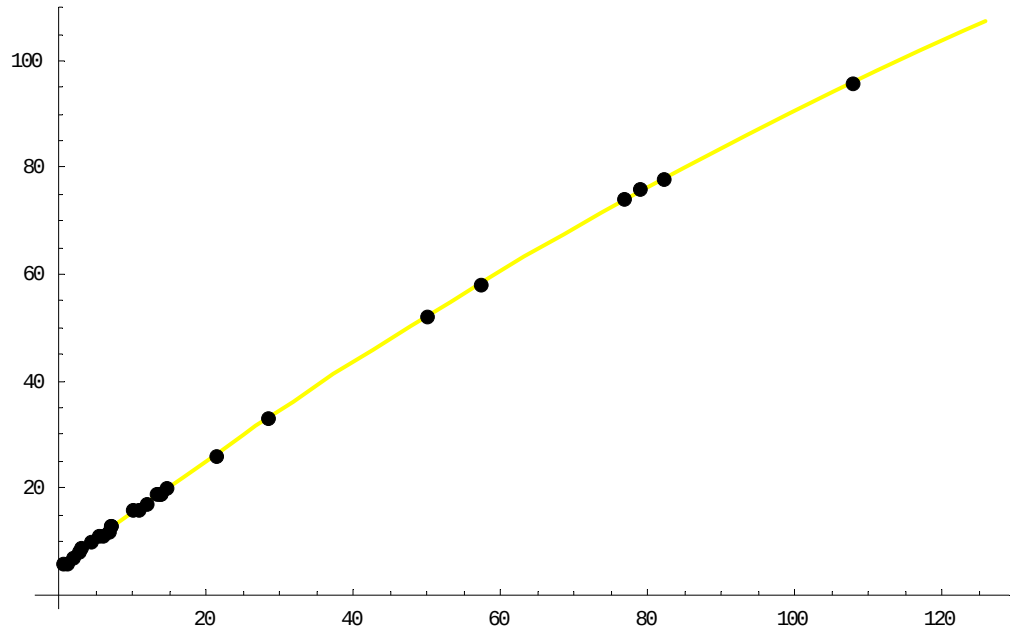

FIGURE 3: Parabolic Allotment EU28 = EU27 + Croatia. Parliament size 750.

3.3 APPLICATION OF THE PARABOLIC METHOD IN OTHER CASES

Let *A* be defined as

$$A(x) = a + bx + cx^2, \quad x \in [m, M].$$

Theorem 1. Providing that the number of States of the EU is at least three (with different populations), for any real value *R*, there are only a few values of the constants *a, b,* and *c,* such that the function *A* of adjustment of quotas of parabolic type verifies that

$$\left.\begin{aligned} A(m) &= 6 \\ A(M) &= 96 \\ \sum_{i=1}^{n} A(q_i) &= R \end{aligned}\right\} \qquad (6)$$

The proof of this result is simple. It suffices to write $A(x)$ in the base $\{1, \quad x-m, \quad (x-m)(M-x)\}$, that is to say, $A(x) = \alpha + \beta(x-m) + \gamma(x-m)(M-x)$, and to force the three conditions (6). Then, the determinant of the system that must be resolved is:

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & M-m & 0 \\ n & H-n*m & z \end{vmatrix} = (M-m)*z > 0$$

The previous determinant is greater than zero, because *z* is positive, *z* being the sum of the values of the function $(x-m)(M-x)$ in the quotas $x = q_i$ of the *n* States; and, therefore, all the addends are positive except for $x = m$ and $x = M$, which are worth zero.

Obviously, since *A*(*x*) changes continuously whenever *R* does, we can increase or diminish *R* to find a parabola that verifies (4). In the case of ties, a unique value of *R* is valid (and only part of the possible rounding are valid, those adding *H*); for other cases, there is an interval of values for *R,* whose solutions verify condition (4). It is possible that the function obtained imposing the previous conditions does not verify any of the other properties described in section 2.2.2.

Consideration

For any value of *R* included between 695 and 890, and the populations given in Table I for the 27 current EU Member States, it is verified that: coefficient *a* is positive, coefficient *b* is positive and lower than the unit, coefficient *c* is negative, and $0 \le A'(x) \le 1, \quad \forall x \in [m, M]$ with which the function of adjustment of quotas *A*, of the parabolic type, verifies all the properties mentioned in Section 2.2.2, the value of *A* even coinciding with the minimal and maximum limitations at the ends of the interval $[m, M]$. This is the current situation, and it will be the future one if there are no important changes in the States comprising the EU.

Theorem 2. Providing that there exist at least three States in the EU (with different populations), for any data (populations, EP size and minimal and maximum limitations) there exists a parabolic function of adjustment of quotas that verifies conditions (1) to (5).

For the proof it is necessary to consider several possibilities, described in the algorithm that is offered below. In all cases it is possible to prove that there is a solution in a very similar way to that demonstrated in theorem 1.

Algorithm to obtain the parabola of quota adjustment

First of all, *A*(*x*) is calculated imposing the conditions:

$$A(m) = 6, \quad A(M) = 96, \quad \sum_{i=1}^{n} \left[A(q_i) \right]_W = H .$$

1. If *A*(*x*) verifies the conditions: (2), (3) and, $A'(q_1) \ge \dfrac{90}{q_n - q_1}$ then $A(x)$ stays definitively as the function of adjustment required, and the algorithm ends. This situation is the one that

happens with the current populations for $H = 750$ (and with any value of H greater than 695 up to 890).

2. If the condition $A'(q_1) \geq \frac{90}{q_n - q_1}$ is not verified, we must recalculate A as the straight line that verifies the 1st condition and in addition $A(m) = 6$. Then the algorithm ends. This situation may appear in the future with an increase in the number of EU member states, in which case Germany would have fewer than 96 deputies.

3. If either $A(x)$ does not increase upon comparison of the assignment of the two largest States, or property 3 is not verified for the two smallest States, we must replace the condition $A(m) = 6$ with $A(m) = k$, choosing a value k greater than 6 but as close as possible to 6, so that the 1st, 2nd and 3rd conditions can be verified. Then the algorithm ends. This situation is very improbable, holding only in the case that many States leave the EU or else the size of the EP increases very substantially.

The rounding to the nearest full number of the exact quotas is what we call a distribution with the parabolic method. A program named MERPE[12] ("Método de Reparto Parabólico de Escaños"), elaborated by J. Martínez-Aroza, a member of our research group) computes the distribution of the seats of the EP using the parabolic method.

4. ANOTHER ADJUSTMENT FUNCTION. THE SQUARE ROOT.

4.1. THE SQUARE ROOT METHOD

One method of allotment to determine the power of the different States in the European Council (EC) that has attracted the attention of researchers on Theory of Games[13,14,15,16,17,18] is the one obtained in proportion to the square root of the population. This distribution is justified in order that every member State has a similar power in the

Ministers Council. We shall now analyse whether this method is degressively proportional in the sense established before.

Of course the mission of the European Parliament is different from that of the Council. Besides, in the EP every State chooses among the political parties that compete; then, these parties train groups of related ideology inside the EP. Therefore, the election of the EP is much more similar to that of the Chamber of Deputies or Representatives of any member State than to the distribution of power in the Council.

On the other hand, on having used a function of adjustment of the type

$$A(x) = K\sqrt{x}\ ,\ \ x \in [m, M\,].$$

Only a parameter "*K*" is obtained for the adjusted quotas, making it impossible to satisfy all properties that we have stated for degressive proportionality.

In fact, for the 750 seats among the 27 current States of the EU, the function $A(x)$ obtained is $A(x) = 6.33885\sqrt{x}$. Table VI shows the adjusted quotas and the completeness with Webster. As can be seen, a distribution of the seats of the EP in proportion to the square root of the population would give an enormous representation to the least populous States.

Table VI. Composition of the EP in proportion to the square root of the population

State	**Population**	**Quota**	**Square Root adjustment**	**Allocation SR**
Malta	404,300	0.62	4.97	**5**
Luxembourg	459,500	0.70	5.30	**5**
Cyprus	766,400	1.17	6.85	**7**

Estonia	1,344,700	2.05	9.07	**9**
Slovenia	2,003,400	3.05	11.07	**11**
Latvia	2,294,600	3.49	11.84	**12**
Lithuania	3,403,300	5.18	14.43	**14**
Ireland	4,209,000	6.40	16.04	**16**
Finland	5,255,600	8.00	17.93	**18**
Slovakia	5,389,200	8.20	18.15	**18**
Denmark	5,427,500	8.26	18.22	**18**
Bulgaria	7,718,800	11.75	21.72	**22**
Austria	8,265,900	12.58	22.48	**23**
Sweden	9,047,800	13.77	23.52	**24**
Hungary	10,076,600	15.33	24.82	**25**
Czech Rep.	10,251,100	15.60	25.04	**25**
Belgium	10,511,400	15.99	25.35	**25**
Portugal	10,569,600	16.08	25.42	**25**
Greece	11,125,200	16.93	26.08	**26**
Netherlands	16,334,200	24.85	31.60	**32**
Rumania	21,610,200	32.88	36.35	**36**
Poland	38,157,100	58.06	48.30	**48**
Spain	43,758,300	66.58	51.73	**52**
Italy	58,751,700	89.40	59.94	**60**
United K.	60,421,900	91.94	60.77	**61**
France	62,886,200	95.69	62.01	**62**

Germany	82,438,000	125.44	71.00	**71**
Total EU-27	492,881,500	750.00	750.00	750

We observe that the exact quota of Ireland surpasses that of Luxembourg in less than 6 seats; and nevertheless the assignment is of 11 seats more for Ireland than to Luxembourg. This contradicts the property 2, the one concerning variation of the quota. Many more comparisons can be observed.

Logically, this is due to the fact that the slope of the fitting curve of the quota is:

$$A'(x) = \frac{6.33885}{2\sqrt{x}}$$

that is, a quantity greater than the unit for any quota less than 10.04 (which corresponds to the least populous States). Then, with the stated criterion underlying this work, the distribution of the seats in proportion to the square root of the population does not produce degressive proportionality for the smallest member States.

Moreover, the adjustment with the square root does not guarantee reaching either of the two limits, since it remains below 6 for Malta and Luxembourg, and below 96 for Germany. Therefore, if it is wished that the smallest States receive at least six seats, we now need to apply Webster with the minimal assignment 6.

Another observation to be underlined is the disproportion between small and big States that the square root method produces. If we calculate for two groups of States, on the one hand the 6 most populous ones, and the 21 remaining ones on the other hand, we arrive at the results shown in Table VII.

Table VII. Unbalance of power in an EP distributed by means of the Square Root method.

States	**Population**	**Population Share**	**Seats Square Root**	**Seats Parabolic**
Germany, France, UK, Italy, Spain, Poland.	346,384,282	70.28 %	354 (47.2%)	438 (58.4%)
Rumania, Netherlands … Luxembourg, Malta	146,468,103	29.72 %	396 (52.8%)	312 (41.6%)

This table makes evident that less than 30 % of the population of the EU would, in theory, elect more than 70 % of the representatives of the EP. It is noteworthy that in light of Table IV, showing the parabolic distribution, the group of the six biggest States receives 438 seats; that is, 70.28 % of the population obtains 58.4 % of the seats if we use the parabolic method.

4.2. OTHER FUNCTIONS

Many other adjustment functions can be used to determinate the composition of the EP. These include power function, spline functions, and other. Some of the spline functions make it quite easy to obtain the allotment, but its concavity is not continuous.

In the paper[19] we offer a further comparison between different methods (parabolic, square root, power, splines, etc.) and different proposals (Lisbon, James-Chopin, etc.) for the Composition of the EP.

5. CONCLUSION

This work describes a proposal presented by two Euro MPs of theCommission of Constitutional Matters, and analyses its implications. It is proven that the concept of degressive proportionality established by the Commission of Constitutional Affaires of the European Parliament *is unacceptable*, provided that sometimes there does not exist a solution verifying such a condition. Therefore, reasonable requirements that a distribution must fulfil with degressive proportionality are stated, and a method (that we call parabolic) for carrying out distributions with degressive proportionality is defined.

Simulations for the distribution of the seats of the EP using the parabolic method have been carried out. In all the cases, we took as populations the ones to be used in order to determine the composition of the EP in the next period; the ones appearing for 2006 in the base http://epp.eurostat.ec.europa.eu. For the simulations we considered the 27 current members of the EU, as well as a size of 750 Euro MPs for the EP, guaranteeing every State a minimum of six seats and a maximum of 96, since all this information appears in the project of The European Constitution. These simulations can be done by means of the simulator MERPE, elaborated by J. Martínez, a member of our research group.

From the simulations performed, it is concluded that the results obtained using the parabolic method do not differ significantly from the results of negotiations carried out in the past in order to determine the distribution of the seats of the EP among the States of the Union.

A distribution of the seats of the EP in proportion to the square root of the population has also been carried out: such a distribution differs substantially from that obtained by using the parabolic method and from the current representation. In addition, it does not fulfil all of the properties that we have established for degressive proportionality, and it greatly over-

represents the smallest States as opposed to the more populous ones. Nevertheless, the distribution in proportion to the square root of the population is a method that fulfils an interesting property in relation with the capacity of power, and therefore, it can prove very convenient for distributing power in other organisations, for example in the Ministers' Council of the European Union, whose circumstances remain beyond the scope of this article.

Indeed, the method based on the square root is supported by most Game Theory specialists worldwide as a means of defining representation in the Council. Let us simply add that it is perfectly justifiable to propose for the EP one method for the distribution of the seats among its members States (for example the parabolic one), and another different method for the Council (for example the square root).

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[1] Draft Treaty establishing a Constitution for Europe. http://european-convention.eu.int/DraftTreaty.asp?lang=EN

[2] F. A. PUKELSHEIM, *A Parliament of degressive representativeness?* Technical Report, preprint Nr.15/2007, Institut für Mathematik, Universität Augsburg.

[3] V. RAMÍREZ, *Some Guidelines for an Electoral European System*, Workshop on Institutions and Voting Rules in the EC, Sevilla (Spain), 2004.

[4] V. RAMÍREZ, A. PALOMARES, M. L. MARQUEZ, *Un método para distribuir los escaños del Parlamento Europeo entre los Estados miembros de la UE*, "Revista Española de Ciencia Política" 2006, 14, pp. 71-85.

[5] V. RAMÍREZ, A. PALOMARES, M.L. MARQUEZ, *Degressively proportional methods for the allotment of the European Parliament seats amongst the UE members Status,* "Mathematics and Democracy", Berlin, Springer Verlag, 2006, pp. 205-220..

[6] V. RAMÍREZ, *The parabolic Method for the allotment of seats in the European Parliament among Members States of the European Union*, Real Instituto Elcano nº 63, 2007.

[7] A. MOBERG, *The Nice Treaty and voting rules in the Council*, "Journal of Common Market Studies" 2002, 40, pp. 259-282.

[8] European Parliament resolution of 11 October 2007 on the composition of the European Parliament (2007/2169(INI)), URL: http://www.europarl.europa.eu/sides/getDoc.do?pubRef=-//EP//TEXT+TA+P6-TA-2007-0429+0+DOC+XML+V0//EN

[9] T. CHOPIN, J-F. JAMET, *The Distribution of MEP seats in the European Parliament between the Member States: both a democratic and diplomatic issue*, Foundation Robert Schuman, "European Issues" 10th September 2007, 71, URL: http://www.robert-schuman.org/question_europe.php?num=qe-71.

[10] M. BALINSKI, H.P. YOUNG, *Fair Representation: Meeting the Ideal of One Man One Vote*, New Haven, CT, 1982.

[11] URL: http://epp.eurostat.ec.europa.eu/portal/page?_pageid=1996,39140985&_dad=portal&

[12] J. MARTÍNEZ, MERPE, URL: http://www.ugr.es/local/vramirez

[13] E. ALGABA, J.M. BILBAO, J.R. FERNANDEZ, *The distribution of power in the European Constitution*, "European Journal of Operational Research" 2007, 176, pp. 1752-1766.

[14] D.S. FELSENTHAL, M. MACHOVER. *Analysis of QM rules in the draft constitution for Europe proposed by the European Convention*, "Social Choice and Welfare" 2003, 23, pp. 1-20.

[15] N. MAASER, S. NAPEL, *Equal representation in two-tier voting systems*, "Social Choice and Welfare" 2007, 28, pp. 401-420.

[16] L.S. PENROSE, *The elementary statistics of majority voting*, "Journal of the Royal Statistical Society" 1946, 109, pp. 53-57.

[17] W. SLOMCZYNSKI, K. ZYCZKOWSKI, *Penrose voting system and optimal quota*, "Acta Physica Polonica" 2006, B 37, pp. 3133-3143.

[18] W. SLOMCZYNSKI, K. ZYCZKOWSKI, *Jagiellonian Compromise an alternative voting system for the Council of the European Union,* Institute of Mathematics and Institute of Physics, Krakow.

[19] J. MARTÍNEZ-AROZA, V. RAMÍREZ, *Comparative analysis of several proposals for the composition f the European Parliament*, preprint.