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## ON BOUNDS FOR ALLOCATION OF SEATS IN THE EUROPEAN PARLIAMENT

### 1. INTRODUCTION

The allocation of seats in the European Parliament between all 27 states forming the Union has to be consistent with restrictions imposed by paragraph 2 of article I-20 of the European Reform Treaty. In particular, the distribution of seats in the future Parliament should obey the following rules:

- i. The number of deputies should not exceed 750;
- ii. The largest state (currently Germany) will obtain at most 96 seats;
- iii. The smallest state (currently Malta) will obtain not fewer than 6 seats;
- iv. The number of seats for each country will be distributed according to the principle of degressive proportionality.

The rules i)-iii) are clear and do not require any comments, but the rule iv) may raise some doubts. Although the very notion of **degressive proportionality** can, in principle, be defined in a mathematically rigorous way, one may construct several different methods of distribution of the seats in the Parliament consistent with this rule. Roughly speaking, the degressive proportionality means a conjunction of two conditions:

- iv.a. Larger state obtains a larger or equal number of seats in the Parliament than a smaller state,

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iv.b. The ratio of the number of seats of a larger and a smaller state is smaller than or equal to the ratio of their populations.

For instance, if the size of a state  $A$  is twice the size of a state  $B$ , then the state  $A$  should obtain more seats in the Parliament than the state  $B$ , but less than twice as much. Usually the size of a state is characterised by the number of its inhabitants or citizens. Alternatively, one may also consider the number of the people eligible to vote - see e.g. Bertini, Gambarelli, Stach (2002, 2005), Pukelsheim (2007a).

Let  $N_i$  and  $N_j$  denote the populations of the states  $i$  and  $j$ , respectively, and let  $S_i$  and  $S_j$  represent the number of their seats in the Parliament. If  $N_i < N_j$ , then

$$\text{a). } S_i \leq S_j ; \quad \text{b). } S_j / S_i \leq N_j / N_i \quad . \quad (1)$$

Condition b) implies that the number of inhabitants per representative in the Parliament increases with the population of a state, namely, if  $N_i < N_j$ , then

$$\text{b'). } N_i / S_i \leq N_j / S_j \quad . \quad (2)$$

In practice, condition b) (or the equivalent condition b') is difficult to be fulfilled, since the number of seats has to be an integer, and as a consequence one encounters the round-off effects. A similar problem appears in the process of allocating parliamentary seats in multiple-winner elections, where each political party gets an integer number of seats and the proportionality principle can be realised only approximately. Thus, it seems reasonable to assume that the principle b) should be also satisfied only in an approximate way. In fact, one can show that there exist such distributions of state populations in a union (Ramirez-Gonzales 2007c) that there is no solution satisfying condition b), see also (Pukelsheim 2007b). Hence, we advocate the following weaker principle of degressive proportionality: the allocation of the seats should satisfy constraint (1) *before* rounding off the non-integer quotas to get the integer number of seats.

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Let us emphasise here that there exist several different systems of allocating seats which satisfy all the necessary requirements i-iv). The literature of the subject contains several reasonable proposals which offer a concrete mathematical representation of the degressive proportionally principle. Although each system has its advantages and drawbacks, it seems rather difficult from a theoretical point of view to distinguish the unique solution of the problem which would be objectively more justified than the other methods proposed.

In this respect, the problem of allocation of the seats in the European Parliament differs considerably from the problem of choosing the system of voting in the European Council. In the latter case, several experts agree that, under natural normative assumption that all potential results of voting are equally likely, the Penrose square root system is objectively distinguished. Namely, it allows us to minimise the democracy deficit (measured by the probability that the decision taken by the Council will not agree with the will of the majority of all citizens of the member states) and, furthermore, it is the only method which gives any citizen of any state the same voting power, measured by the composed Penrose-Banzhaf index (which is the product of the power indices of a citizen voting in his country and of the representative of his state voting in the Council).

The key difference between both problems results from the fact that a representative of a member state in the Council cannot split his votes. Even if only 51% of the population of the country supports a given decision, he has to vote 'yes' on behalf of the total population. This very rule leads to the system in which the votes of a given country are proportional to the square root of its population – see e.g. Słomczyński and Życzkowski 2006 and 2007. On the other hand, the representatives of a country in the European Parliament can split their votes in order to represent optimally the opinion of citizens who had elected them. Thus, application of the square root weights in designing a system of distribution of the seats in the Parliament

does not seem to be justified. Furthermore, the voting power of a single state in the Parliament depends not only on the number of its representatives, but also on their distribution between various political fractions in the Parliament (see Fedeli, Forte 2002), which cannot be predicted *a priori*.

## 2. REGULAT METHODS OF ALLOCATION OF SEATS

A mathematical approach to the problem of allocating seats in the Parliament can be described by the following scheme. First, one needs to choose a concrete characterisation of the size of a given state  $i$  by a number  $N_i$  (e.g., equal to the total number of its inhabitants, citizens, or voters), and precisely define by which means these data are collected and how often they should be updated. Then, one needs to transform the data by a suitable function  $f$ , which:

- I. is **non-decreasing**;      II. is **concave**.

In this way one obtains the **quota**  $Q_i$  for the  $i$ -th member state:

$$Q_i = f(N_i) \tag{3}$$

It is easy to see that thus obtained real numbers  $Q_i$  satisfy conditions (1a) and (1b). In the last step one distributes 750 seats in the Parliament proportionally to the quota using an appropriate allocation method, e.g., the Webster method (equivalent to the Sainte-Laguë method). The numbers  $S_i$  obtained in this way provide the desired allocation of seats. Note that for any state the integer number  $S_i$  is proportional, up to inevitable round-off errors, to the quota  $Q_i$ . The distribution of seats obtained in this way fulfils the conditions i., ii., iii. and iv.a. However, the condition iv.b. can be violated in some cases due to the round-off procedure.

Any method of allocating seats constructed in this way, and depending only on the function  $f$ , will be called **regular**. Practical implementation of such methods consists hence in selecting an appropriate non-decreasing and concave function  $f$ . However, from a mathematical point of view there exist infinitely many such functions, and, consequently, there exist many different regular methods of allocating seats in the Parliament which fulfil the criteria i.-iv.

To simplify the problem it is natural to consider only a certain class of non-decreasing and concave functions, e.g. these which depend on three parameters ( $a$ ,  $b$  and  $c$ ). Then, their values can be set by the requirements that the number of seats for the smallest state is equal to 6, the largest 96, and their sum equals 750. Again, there exist a lot of such three parameter families, so it is difficult to find strong objective arguments in favour of a concrete choice.

For instance one can consider the following classes:

$$\text{- parabolic} \quad f(x) = a + bx - cx^2 \quad (4)$$

$$\text{- linear-hyperbolic} \quad f(x) = a + bx - \frac{c}{x} \quad (5)$$

$$\text{- power law} \quad f(x) = a + bx^c \quad (6)$$

### 3. LOWER AND UPPER BOUND FOR THE NUMBER OF SEATS FOR EACH MEMBER STATE

Let us assume that for a regular system of allocating seats the smallest and the largest state are given 6 and 96 seats, respectively. This natural assumption joined with the monotonicity and concavity conditions allows us to derive lower and upper bounds for the number of seats  $S_i$  for any of the  $M$  member states of the union:

$$S_i^{\min} \leq S_i \leq S_i^{\max} \quad \text{for } i = 1, \dots, M. \quad (7)$$

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The exact value of a lower bound is obtained by assigning the smallest and the largest state their prescribed values and applying the linear interpolation. The number  $S_i^{\min}$  obtained in this way determines a 'reasonable' minimum for the number of seats each state should receive. In a similar manner one obtains an upper bound by constructing a piecewise linear function, such that both linear segments are attached to the fixed extreme points (1,6) and (M,96), while the gluing point  $(i, S_i^{\max})$  is determined for each state by the normalisation constraint  $(\sum_{i=1}^M S_i = 750)$  – see Fig.1. Both bounds obtained in this way for EU-27 are collected in

Table 1. In our opinion **the arithmetical mean  $S^{\text{mean}} = (S^{\min} + S^{\max}) / 2$  of the bounds can serve as a reference line to gauge other allocation schemes.**

Member State	M	Population (in S)	Percentage of the	$S^{\min}$	$S^{\max}$	$S^{\text{mean}}$
Germany	D	82.438	16.73%	<b>96</b>	<b>96</b>	<b>96</b>
France	FR	62.886	12.76%	<b>74</b>	<b>85</b>	<b>79,5</b>
United Kingdom	U	60.422	12.26%	<b>71</b>	<b>82</b>	<b>76,5</b>
Italy	IT	58.752	11.92%	<b>70</b>	<b>80</b>	<b>75</b>
Spain	ES	43.758	8.88%	<b>53</b>	<b>63</b>	<b>58</b>
Poland	PL	38.157	7.74%	<b>47</b>	<b>57</b>	<b>52</b>
Romania	R	21.610	4.38%	<b>29</b>	<b>36</b>	<b>32,5</b>
Netherlands	N	16.334	3.31%	<b>23</b>	<b>30</b>	<b>26,5</b>
Greece	EL	11.125	2.26%	<b>17</b>	<b>23</b>	<b>20</b>
Portugal	PT	10.570	2.14%	<b>17</b>	<b>22</b>	<b>19,5</b>
Belgium	B	10.511	2.13%	<b>17</b>	<b>22</b>	<b>19,5</b>
Czech	C	10.251	2.08%	<b>16</b>	<b>21</b>	<b>18,5</b>
Hungary	H	10.077	2.04%	<b>16</b>	<b>21</b>	<b>18,5</b>
Sweden	SE	9.048	1.84%	<b>15</b>	<b>20</b>	<b>17,5</b>
Austria	A	8.266	1.68%	<b>14</b>	<b>19</b>	<b>16,5</b>
Bulgaria	B	7.719	1.57%	<b>14</b>	<b>18</b>	<b>16</b>
Denmark	D	5.428	1.10%	<b>11</b>	<b>15</b>	<b>13</b>
Slovakia	S	5.389	1.09%	<b>11</b>	<b>15</b>	<b>13</b>
Finland	FI	5.256	1.07%	<b>11</b>	<b>15</b>	<b>13</b>
Ireland	IE	4.209	0.85%	<b>10</b>	<b>14</b>	<b>12</b>
Lithuania	LT	3.403	0.69%	<b>9</b>	<b>13</b>	<b>11</b>
Latvia	L	2.295	0.47%	<b>8</b>	<b>11</b>	<b>9,5</b>

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Slovenia	SI	2.003	0.41%	<b>7</b>	<b>11</b>	<b>9</b>
Estonia	EE	1.344	0.27%	<b>7</b>	<b>10</b>	<b>8,5</b>
Cyprus	C	0.766	0.16%	<b>6</b>	<b>10</b>	<b>8</b>
Luxembourg	L	0.460	0.09%	<b>6</b>	<b>9</b>	<b>7,5</b>
Malta	M	0.404	0.08%	<b>6</b>	<b>6</b>	<b>6</b>
EU-27	E	492.881	100.00%			

Table 1. Lower bound  $S^{\min}$  and upper bound  $S^{\max}$  for the number of seats in the European Parliament for each of 27 members of the European Union. First columns provide the name of each state, its abbreviation, the population in millions set by Commission on Nov 7, 2006 (see Doc. 15124/06) according to the EUROSTAT's data, the percentage of the population of a given state with respect to the total population of the Union.

#### 4. THE LAMASSOURE-SEVERIN PROJECT

The authors of the recent report (Lamassoure, Severin 2007) prepared for the European Parliament based their allocation scheme on some seemingly natural postulates.

They are closely related to conditions i.-iv. formulated in Sect. 1:

- *rule 17a* – corresponds to condition i.;
- *rule 17b* – corresponds to condition ii.;
- *rule 17c* – is not mentioned in the report (?);
- *rules 17d-17f* – aim to express the principle of ‘degressive proportionality’. In particular:
  - *rule 17f* – corresponds to condition iv.a.;
  - *rule 17e* – corresponds to condition iv.b.

On the other hand, *rule 17g* has a slightly different character. It is not formulated in a clear way, so in practise it may be interpreted by the members of the Parliament in various ways. Additionally, in paragraph 17. the authors supply an extra rule which states that the new number of seats allotted to each state should not be smaller than the current one following from the rules accepted in the protocol concerning the admission of Bulgaria and Romania to

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the Union. This practical rule does not follow from the principle of degressive proportionality, nor from any other article of the Reform Treaty. It is then easy to see that the acceptance of this implicit rule implies that any irregularities of the current compromise will persist, sometimes even in an enhanced way, in the new solution.

The authors of the report do not provide an explicit algorithm describing how to allocate the seats. A careful mathematical analysis of their proposal suggests that their system is a kind of an *ad hoc* solution of the current problem, not based on a concrete mathematical model. Thus, it is not possible to use their scheme of allocation of the seats in future, e.g. to take into account possible demographic changes or admitting new member states to the Union.

Furthermore, Lamassoure and Severin aim to rigorously fulfil the rule that the ratio of the number of seats to the population in each state,  $S_i / N_i$ , decreases with the population of the state, although it is known (Ramirez-Gonzales 2007c) that for certain distribution of the population such a solution is not possible. As argued in a previous section, this ratio should be treated as strictly monotonous only before the round-off procedure is done. Requiring monotonicity of this ratio computed for integer numbers of seats for each state results in several irregularities of the allocation scheme obtained. It is especially visible, if we compare the Lamassoure and Severin proposal with the bounds computed in Sect. 3. We see that their proposal:

- a) attributes too many seats to the medium size states (e.g. to Austria, Sweden, Hungary, Czech Republic, but also to Bulgaria, Belgium, and Portugal);
- b) gives too few seats to large states (in particular to France) and to some small states (in particular to Estonia);

- c) in two cases (Czech Republic and Hungary) the number of seats is larger than the upper bound, while for Estonia it is smaller than the lower bound.

In spite of this obvious deficiencies, the Lamassoure and Severin proposal was accepted by the Committee on Constitutional Affairs of the European Parliament, and then by the Intergovernmental Conference in Lisbon (18 October 2007) with a small modification (one additional seat for Italy).

## 5. OTHER METHODS ADVOCATED IN THE LITERATURE

### A. THE RAMIREZ-GONZALEZ PARABOLIC REGULAR METHOD

This regular method was proposed by Ramírez González and his co-workers in a series of recent papers (Ramirez-Gonzales 2004, Ramirez-Gonzales, Palomares Bautista, Marquez Garcia 2006a and 2006b, Ramirez-Gonzales 2007a). The principle of 'degressive proportionality' is realised by the parabolic function (4), where  $x$  stands for the population of a state and  $f$  denotes the approximate number of seats, which is eventually allocated by the Webster method. Taking into account the necessary constraints, one obtains the values of the three parameters:  $a = 5.44132$ ,  $b = 1.38428$ , and  $c = 0.0034665$ . Such a solution is mathematically appealing as it uses a natural concave function and fulfils all the constraints required. Although this method is not uniquely distinguished among other regular methods of allocating seats, it is conceptually simple and easy to apply in the case of further extensions of the Union. Note that this solution is very close to the arithmetical mean of the lower and upper bounds for regular methods computed in Sect. 3.

## B. LINEAR-HYPERBOLIC REGULAR METHOD

Linear-hyperbolic regular method proposed in this work is a regular method based on the hyperbolic function (5). The optimal values of the fitting parameters read  $a = 9.01655$ ,  $b = 1.05534$ , and  $c = 1.39093$ . This proposal yields results which are advantageous to small and medium size states.

## C. POWER REGULAR METHOD

Power regular method is a regular method based on the power formula (6). It provides a natural interpolation between the flat distribution (the same number of seats for every state, like in the US Senate) and the linear distribution. Constraints imply optimal values of the fitting parameters,  $a = 5.12405$ ,  $b = 1.93203$  i  $c = 0.87282$ , see also Martinez-Aroza and Ramirez-Gonzales 2007.

## D. THE PUKELSHEIM QUASI-PROPORTIONAL METHODS

Three variants of the proportional system of allocating seats are investigated in a work of Pukelsheim (Pukelsheim 2007a). As a starting point he takes the number of people eligible to vote in each state during the recent European Parliament election in 2004, which is not directly proportional to the total population. For instance, Italy with 49.8 million is the second largest state after Germany if number of voters is considered although, concerning its total population, this country is the fourth largest in the EU, after France and the United Kingdom which have 44.1 and 41.5 million of voters respectively. Since, due to the constraints i.-iii., the strict proportionality in allocation of the seats cannot be fulfilled, Pukelsheim considers three variant solutions, which fulfil all required constitutional conditions:

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Variant A) **Restricted proportionality** is the simplest realisation of proportionality: the number of seats grows proportionally to the number of voters, with constraints for the number of seats for the 'extreme' states. Such a system increases power of the largest states.

Variant B) **Stratified proportionality** divides all member states into two groups: the small states obtain jointly 250 seats, while the largest obtain remaining 500 seats. In both groups the distribution of seats is strictly proportional, up to the round-off errors.

Variant C) **Deferred proportionality**: each state obtains the minimum number of 6 seats and the remaining seats are allocated proportionally. This simple method provides reasonable results, and the system seems to be politically more balanced than two other variants. In general, such a linear interpretation of the 'degressive proportionality' is advantageous for the largest states at the expense of medium states.

In Pukelsheim analyses (Pukelsheim 2007b) a modification of Variant C (called '**Fix + Prop.**') based on the total populations instead of the number of people eligible to vote.

#### E. THE TAAGEPERA-HOSLI LOGARITHMIC METHOD

This method was proposed by Taagepera and Hosli (Taagepera, Hosli 2006) and may be applied both to solve the problem of allocating seats in the European Parliament and distribution of votes in the EU Council. The first step lies in setting the value of the parameter (exponent)  $n$  as:

$$n = (1/\log K - 1/\log S) / (1/\log K - 1/\log N) ,$$

where:

$K$  – the number of member states,

$S$  – the number of seats in the Parliament,

$N$  – the total population of the EU.

Then one determines the number of seats  $S_i$  in the EP for the  $i$ -th country as (after necessary rounding-off):

$$S_i = S \cdot N_i^n / \sum_{i=1}^K N_i^n ,$$

where  $N_i$  is the population of the  $i$ -th state.

We have applied this method to the current data and put the results in Tab. 2. It seems, however, that the Taagepera – Hosli solution gives too many seats to the medium states, and too few to the large ones to be seriously considered by politicians.

#### F. OTHER METHODS

Let us mention here yet other possible solutions based on a very different assumptions. The method proposed by Bertini, Gambarelli, and Stach in [BGS02, BGS05] takes as a starting point not only the population of a given state, but also its economic power measured by the gross domestic product. Taking a suitable combination of these two factors one can look for a solution which takes into account both of them. Such a method is particularly advantageous for small and medium size states with a highly developed economy, like the Netherlands. On the other hand, recent proposal of the Robert Schuman Foundation (Chopin, Jamet, 2007) is not based on any clear mathematical formula. A comparison of the results obtained with various methods is presented in Tab 2 and Fig. 1. and 2.

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Member state	Co de	Populatio n (in millions)	% of EU-27 populati on	Seats till 2009	Seats (2009- 2014)	Parabo lic Ramír ez	Linear – hyperb olic	Power	Varian t C Pukels heim	Fix + Prop. Pukels heim	Hosli Taagep era	Min	Max	Mea n
Germany	DE	82,438	16,73%	99	96	96	96	96	96	96	81	96	96	96
France	FR	62,886	12,76%	78	74	79	76	77	85	83	69	74	85	79,5
United Kingdom	UK	60,422	12,26%	78	73	76	73	74	76	80	67	71	82	76,5
Italy	IT	58,752	11,92%	78	73	75	71	73	71	77	66	70	80	75
Spain	ES	43,758	8,88%	54	54	59	55	57	61	59	55	53	63	58
Poland	PL	38,157	7,74%	54	51	53	49	51	53	52	51	47	57	52
Romania	RO	21,610	4,38%	35	33	34	32	33	34	32	36	29	36	32,5
Netherlands	NL	16,334	3,31%	27	26	27	26	27	25	26	31	23	30	26,5
Greece	EL	11,125	2,26%	24	22	20	21	21	22	20	24	17	23	20
Portugal	PT	10,570	2,14%	24	22	20	20	20	20	19	24	17	22	19,5
Belgium	BE	10,511	2,13%	24	22	20	20	20	19	19	23	17	22	19,5

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Czech Rep.	CZ	10,251	2,08%	24	22	19	20	20	19	18	23	16	21	18,5
Hungary	HU	10,077	2,04%	24	22	19	20	20	18	18	23	16	21	18,5
Sweden	SE	9,048	1,84%	19	20	18	18	18	17	17	21	15	20	17,5
Austria	AT	8,266	1,68%	18	19	17	18	17	16	16	20	14	19	16,5
Bulgaria	BG	7,719	1,57%	18	18	16	17	17	16	15	20	14	18	16
Denmark	DK	5,428	1,10%	14	13	13	15	14	13	13	16	11	15	13
Slovakia	SK	5,389	1,09%	14	13	13	14	14	13	13	16	11	15	13
Finland	FI	5,256	1,07%	14	13	13	14	13	12	12	15	11	15	13
Ireland	IE	4,209	0,85%	13	12	11	13	12	11	11	14	10	14	12
Lithuania	LT	3,403	0,69%	13	12	10	12	11	10	10	12	9	13	11
Latvia	LV	2,295	0,47%	9	9	9	11	9	9	9	9	8	11	9,5
Slovenia	SI	2,003	0,41%	7	8	8	10	9	8	8	9	7	11	9
Estonia	EE	1,344	0,27%	6	6	7	9	8	7	8	7	7	10	8,5
Cyprus	CY	0,766	0,16%	6	6	6	8	7	7	7	6	6	10	8
Luxembourg	LU	0,460	0,09%	6	6	6	6	6	6	7	6	6	9	7,5
Malta	MT	0,404	0,08%	5	6	6	6	6	6	6	6	6	6	6

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EU-27	EU	492,881	100,00 %	785	751	750	750	750	750	751	750			
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Table 2. Allocation of seats in the European Parliament according to several proposals. The columns contain: (1) the name of a member state; (2) the code of a state; (3) population figures as officially established on 7 November 2006 by the Commission in Doc. 15124/06 on the basis of *EUROSTAT* figures; (4) the percentage of the EU-27 population; (5) the current distribution of seats in the European Parliament valid until 2009; (6) the number of seats for the period 2009-2014 as stated by the IGC 2007; the distribution of seats according to: (7) the parabolic regular method of Ramírez *et al.* (Ramírez-Gonzales 2004, Ramírez-Gonzales, Palomares Bautista, Marquez Garcia 2006a and 2006b, Ramírez-Gonzales 2007a); (8) the linear-hyperbolic regular method, (9) the power regular method, (10) the Puckelsheim linear method (variant C - deferred proportionality) [Pu07a], (11) the Puckelsheim linear method ('Fix + Prop.')[Pu07b], (12) the Taagepera-Hosli logarithmic method (Taagepera, Hosli 2006); (13-15) the lower ( $S^{\min}$ ) and the upper ( $S^{\max}$ ) bounds, and their arithmetical mean ( $S^{\text{mean}}$ ).

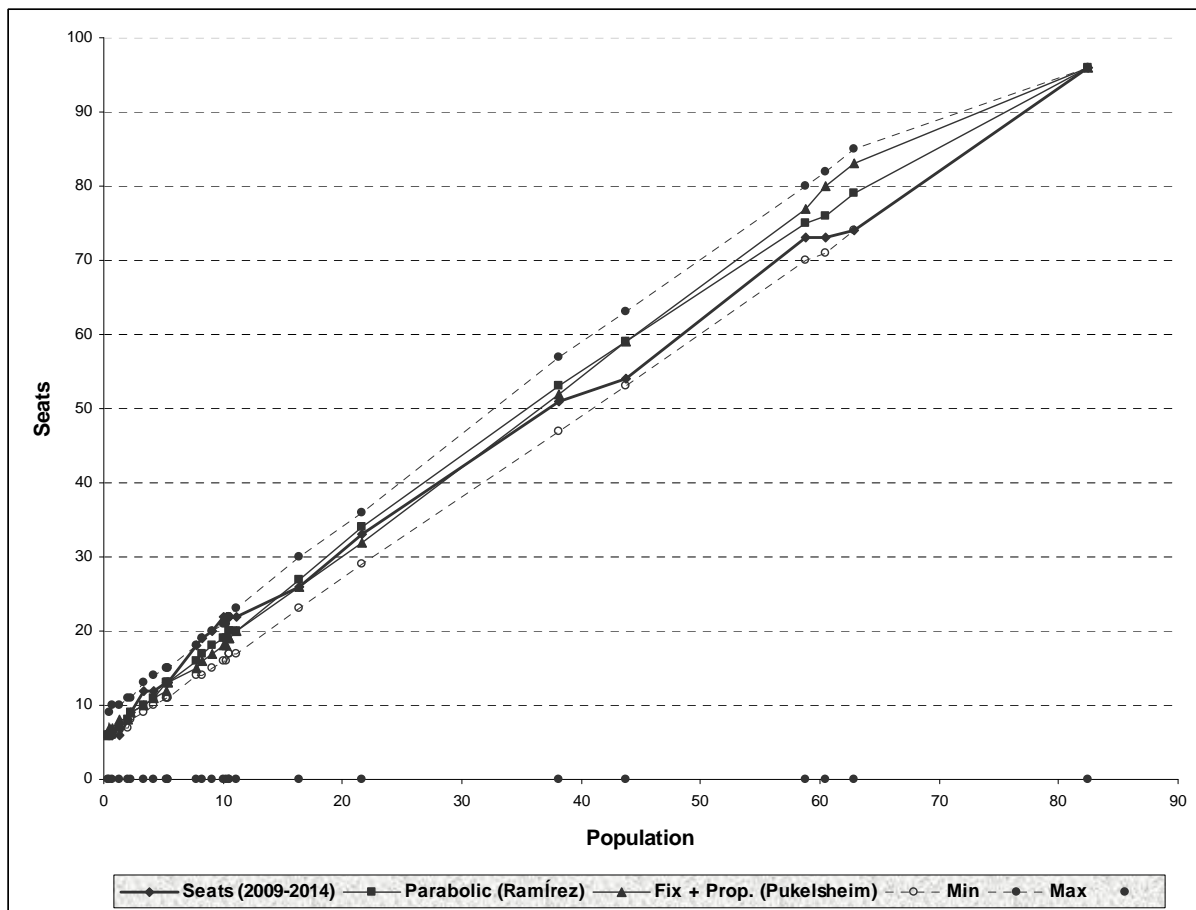


Fig. 1. Allocation of seats in the European Parliament (all countries): the number of seats for the period 2009-2014 as stated by the IGC 2007; the distribution of seats according to: the parabolic regular method of Ramírez *et al.*; the Puckelsheim liner method ('Fix + Prop.');

the lower ( $S^{\min}$ ) and the upper ( $S^{\max}$ ) bounds.

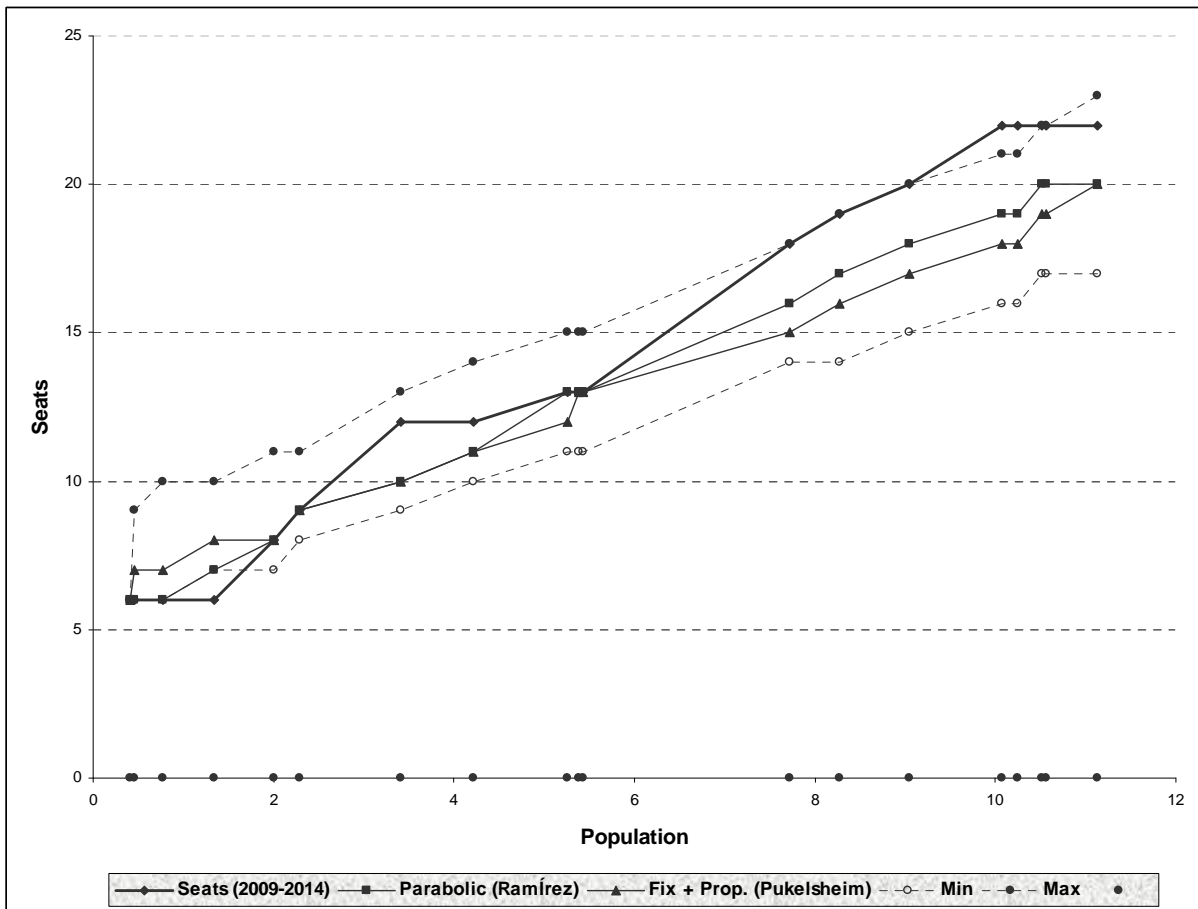


Fig. 2. Allocation of seats in the European Parliament (countries from Malta to Greece): the number of seats for the period 2009-2014 as stated by the IGC 2007; the distribution of seats according to: the parabolic regular method of Ramírez *et al.*; the Puckelsheim linear method ('Fix + Prop. '); the lower ( $S^{\min}$ ) and the upper ( $S^{\max}$ ) bounds.

## 6. CONCLUDING REMARKS

- i) Any system of allocating seats in the European Parliament should obey the principle of 'degressive proportionality'. Although this principle can be made mathematically rigorous, it does not lead to a unique solution. Hence, it is possible to construct several different allocation systems which satisfy all required constitutional constraints.
- ii) Theoretic analysis does not allow one to distinguish an optimal system of allocation of the seats in the European Parliament.

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- iii) The Penrose square root system, optimal in the case of the European Council, is not distinguished by any arguments in the case of the Parliament. The reason is that the representative of each state cannot split his vote in the Council, while members of the Parliament from a single country may vote differently to represent the opinion of their electors.
- iv) The Severin-Lamassour system is not based on any clear theoretical arguments. It is an *ad hoc* solution which cannot be applied in the case of future extensions of the Union. Several states including both the small ones (Estonia) and the large ones (e.g. France) are evidently handicapped by this proposal.
- v) From the mathematical point of view one may distinguish **regular methods** of allocating seats in the European Parliament which realise the principle of degressive proportionality and are based on a use of particular non-decreasing and concave function.
- vi) On the basis of the constraints following from the Reform Treaty, we compute the upper and lower bounds for the number of seats allocated to each state in a regular method. **The average of these numbers can provide a reference line to gauge other possible allocation schemes.**
- vii) The **parabolic method** of Ramírez *et al.* is regular and gives the results close to the average of the lower and upper bounds for regular methods.
- viii) As regards other methods of allocating seats, the Pukelsheim **deferred proportionality method** is based on solid mathematical arguments and seems to be politically realistic. It also gives the results which lie within the lower and upper bounds for regular methods.

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